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NUMERICAL SOLUTIONS OF AN EIGENVALUE PROBLEM IN UNBOUNDED DOMAINS*

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Abstract *A coupling method of finite element and infinite large element is proposed for the numerical solution of an eigenvalue problem in unbounded domains in this paper. With some conditions satisfied, the considered problem is proved to have discrete spectra. Several numerical experiments are presented. The results demonstrate the feasibility of the proposed method.*

Key words *eigenvalue problem, unbounded domain, infinite large element method.*

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1 Introduction

A great deal of work has been done on the numerical solution of eigenvalue problems, which have widespread applications in physics and engineering. In this paper, we will consider an eigenvalue problem in unbounded domains and introduce a numerical approach for the proposed problem.

This paper is inspired by the successful application of infinite large element to Helmholtz equation in exterior domains by K.Gerds[7,8,9] and L.Demkowicz[9] and various concepts of large element and infinite large elements developed by Han Hou-de and Ying Lung-an[2], P.Bettess[4,5], and D.S.Burnett[6]. Here we will introduce a coupling method of finite element(FE) and infinite large element(ILE) to overcome the essential difficulty for obtaining the numerical solution of the given problem which originates from the unboundedness of physical domain.

We now consider the following eigenvalue problem in the unbounded domain Ω^e

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Find $\lambda \in \mathbb{C}, u \neq 0$ such that

$$-\triangle u - \lambda\rho(x)u = 0, \qquad \text{in } \Omega^e, \tag{1.1}$$

$$u|_\Gamma = 0, \qquad \text{on } \Gamma, \tag{1.2}$$

$$\int_{\Omega^e} |\nabla u|^2 \mathrm{d}x + \int_{\Omega^e} \rho |u|^2 \mathrm{d}x < \infty. \tag{1.3}$$

Here $\Omega = \mathbb{R}^2 \backslash \bar{\Omega}^e$ is assumed to be a bounded domain with smooth boundary Γ. $\rho(x) > 0$ is continuous in $\mathbb{R}^2$. Besides, we assume

$$B = \{x : |x| < 1\} \subset\subset \Omega.$$

Problem (1.1)-(1.3) can be deduced from the following initial-boundary of heat equation on the unbounded domain $\Omega^e \times (0, T]$:

$$\rho(x)\frac{\partial w}{\partial t} = \triangle w, \quad (x,t) \in \Omega^e \times (0,T], \tag{1.4}$$

$$w|_\Gamma = 0, \qquad 0 < t \leq T, \tag{1.5}$$

$$w|_{t=0} = w_0(x), \tag{1.6}$$

$$w(x,t) \ \text{ is bounded}, \tag{1.7}$$

where $w_0(x)$ is a given function, $w|_\Gamma = 0$ and support $\{w_0(x)\}$ is compact. Consider the solution of the problem (1.4)-(1.7) in the following form

$$w(x,t) = e^{-\lambda t} u(x), \tag{1.8}$$

where $(\lambda, u(x))$ is to be determined. Substituting (1.8) into problem (1.4)-(1.7) we know that $\{\lambda, u(x)\}$ is determined by eigenvalue problem (1.1)-(1.3).

The organization is as the following. In section 2, we introduce the variational formulation of the eigenvalue problem and analyze some of its properties. In section 3, we present the discretization of exterior domain. Some numerical examples are given in section 4.

2 Some properties of the eigenvalue problem

We suppose $\rho(x)$ satisfies:

$$0 < \delta(R) \leq \min_{1 \leq |x| \leq R} \rho(x), \quad \text{for} \quad R \geq 1, \tag{2.1}$$

$$\rho(x) \leq \rho_0(|x|), \quad 1 \leq |x| < +\infty, \tag{2.2}$$

$$M = \int_1^\infty r \ln r \rho_0(r) \mathrm{d}r < +\infty,$$