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Solvability of a Elliptic-Parabolic (Hyperbolic) Type Chemotaxis System in a Bounded Domain

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Abstract. In this paper, we use contraction mapping principle and operator-theoretic approach to establish local solvability of the elliptic-parabolic(hyperbolic) Type Chemotaxis System. In addition, global solvability of the systems is considered by some uniform estimates.

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1 Introduction

Isolated from decaying leaves collected in a hardwood forest of the North Carolina mountains in the summer of 1933, the cellular slime mold Dictyostelium discoideum was discovered by K. B. Raper in 1935 [1]. In those days it was hard for K. B. Raper to imagine, that more than many years later this discovery would have attracted a large group of mathematicians [2–12] to lay their scientific focus on a model proposed by E. F. Keller and L. A. Segel [13] in 1970 to describe the aggregation phase of the Dictyostelium discoideum.

V. Nanjundiah [14] considered the following steady state system:

$$\begin{aligned} &\nabla(\nabla u - u\nabla v) = 0 \qquad && \text{in } \Omega\times(0,T),\\ &\Delta v - v + u = 0 && \text{in } \Omega\times(0,T), \end{aligned}$$

where

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- $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$;
- $u = u(x,t)$ is an unknown function of $(x,t) \in \Omega \times [0,T)$ and it stands for the density of cellular slime molds;
- $v = v(x,t)$ is an unknown function of $(x,t) \in \Omega \times [0,T)$ and it stands for the concentration of chemical substances secreted by the slime molds.

The null-flux boundary condition is

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0 \quad \text{on } \partial\Omega \times (0,T),$$

where n is the outer unit vector on $\partial\Omega$. The initial condition is given as

$$v|_{t=0} = v_0(x) \quad \text{on } \Omega,$$

where $v_0(x)$ is a given function.

Motivated by it, we consider the following two systems:

$$\begin{cases} \nabla\left[\nabla u - u\nabla v\right] = ve^v & \text{in } \Omega \times (0,T), \\ v_t = \Delta v - v + ue^{-v} & \text{in } \Omega \times (0,T), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega \times (0,T), \\ v\big|_{t=0} = v_0(x) & \text{in } \Omega, \end{cases} \tag{1.1}$$

and

$$\begin{cases} \nabla\left[\nabla u - u\nabla v\right] = ve^v & \text{in } \Omega \times (0,T), \\ v_{tt} = \Delta v - v + ue^{-v} & \text{in } \Omega \times (0,T), \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega \times (0,T), \\ v\big|_{t=0} = v_0(x) & \text{in } \Omega \\ v_t\big|_{t=0} = v_1(x) & \text{in } \Omega. \end{cases} \tag{1.2}$$

This paper is arranged as follows:

In Section 2, we give the main results and recall some properties of the Laplacian in Ω supplemented with homogeneous Neumann boundary condition. In Section 3, making use of contraction mapping principle and operator-theoretic approach, we will prove the local existence of system (1.1) in Sobolev space. In Section 4, the global existence of system (1.1) is considered by Gronwall's inequality. In addition, the local and global existence of system (1.2) is established in Section 5.