Seiberg-Witten-Like Equations Without Self-Duality on Odd Dimensional Manifolds

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Abstract. In this paper, Seiberg-Witten-like equations without self-duality are defined on any smooth $2n+1$-dimensional Spin$^c$ manifolds. Then, a non-trivial solution is given on the strictly-Pseudoconvex CR-5 manifolds endowed with a canonical Spin$^c$-structure by using Dirac operator associated with the generalized Tanaka-Webster connection. Finally, some bounds are given to them on the 5-dimensional Riemannian manifolds.

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1 Introduction

Recently, Seiberg-Witten theory has played an important role in the topology of 4-manifolds. One of the important part of this theory is Seiberg-Witten equations consist of curvature equation and Dirac equation. Dirac equation can be written down on any Spin$^c$-manifold in any dimension. Due to the self-duality of a 2-form, the curvature equation is special to 4-dimensional manifolds. There are some generalizations of these equations to higher dimensional manifolds. All of them are mainly based on the generalized self-duality of a 2-form [1, 2].

A global solution to these equations is given by means of the canonical Spin$^c$-structure. Since any almost Hermitian manifold has a canonical Spin$^c$-structure which is determined by its almost Hermitian structure, a fundamental role is played by the almost Hermitian manifold. As in almost Hermitian manifolds, every contact metric manifold

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can be equipped with the canonical Spin^c-structure determined by the almost complex structure on the contact distribution. Also, on these manifolds, one can described a spinor bundle. Therefore, for a given canonical Spin^c-structure on the contact metric manifold, a spinorial connection can be defined on the associated spinor bundle by means of the generalized Tanaka-Webster connection [3]. Then, on any contact metric manifolds endowed with Spin^c-structure, Dirac operator is associated to such connection. Also, Kohn-Dirac operator is defined by restriction of the contact metric manifold to the contact distribution. By using this Dirac operator we define Dirac equation on a 2n+1-dimensional contact metric manifold. Also, curvature equation is written down without using self-duality concept on any (2n+1)-dimensional contact metric manifold via orthogonal projection of the endomorphisms onto a particular subbundle.

The plan of this paper is in the following. In Section 2, some basic facts concerning the contact metric manifolds and CR manifolds are written. Then canonical Spin^c-structure and Dirac operator are defined by means of the generalized Tanaka-Webster connection. In Section 3, Seiberg-Witten-like equations without self-duality are defined on (2n+1)-dimensional contact metric manifold by means of the generalized Tanaka-Webster connection. Then, in particular, a non-trivial solution is given on the 5-dimensional strictly-Pseudoconvex CR manifolds. In the final section, some bounds are given to the solution of these equations on the 5-dimensional Riemannian manifolds.

2 Some basic materials

2.1 Spin^c manifolds

Assume that M is an 2n+1 dimensional orientable Riemannian manifold. Then the definitions of the Spin^c-structures on M are obtained as follows:

Accordingly, the structure group of M is SO(2n+1) and there is an open covering \{U_\alpha\}_{\alpha \in \Lambda} with the transition functions \(g_{\alpha\beta}: U_\alpha \cap U_\beta \rightarrow SO(2n+1)\) for M. In addition, if there exists another collection of transition functions \(\tilde{g}_{\alpha\beta}: U_\alpha \cap U_\beta \rightarrow SO(2n+1)\) such that the following diagram commutes

\[
\begin{array}{ccc}
U_\alpha \cup U_\beta & \xrightarrow{g_{\alpha\beta}} & SO(2n+1) \\
\xrightarrow{\tilde{g}_{\alpha\beta}} & & \\
& \lambda \downarrow & \\
& SO(2n+1) &
\end{array}
\]

that is, \(\lambda \circ \tilde{g}_{\alpha\beta} = g_{\alpha\beta}\) and the cocycle condition \(\tilde{g}_{\alpha\beta}(x) \circ \tilde{g}_{\beta\gamma}(x) = \tilde{g}_{\alpha\gamma}(x)\) on \(U_\alpha \cap U_\beta \cap U_\gamma\) is satisfied, then M is called Spin^c manifold. Then on a Spin^c manifold M, one can construct three principal bundles [4].