JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **30**, No. 4, pp. 329-343

doi: 10.4208/jpde.v30.n4.4
December 2017

Hyperbolic-Parabolic Type Chemotaxis Systems in $\mathbf{R}^{\mathbf{N}}$

WU Shaohua and ZHOU Kai*

School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China.

Received 26 February 2017; Accepted 31 October 2017

Abstract. In this paper, we discuss the local and global existence of weak solutions for some hyperbolic-parabolic systems modeling chemotaxis in $\mathbf{R}^{\mathbf{N}}$.

AMS Subject Classifications: 35A07, 35K50, 35M10, 35L10

Chinese Library Classifications: O175.29

Key Words: Hyperbolic-parabolic system; KS model; chemotaxis.

1 Introduction

The KS model, which is described by the following parabolic system

$$u_t = \nabla(\nabla u - \chi(v)\nabla v\cdot u), \qquad \text{in}\,(0,\infty)\times\Omega, \tag{1.1}$$

$$\tau v_t = \Delta v + g(v,u), \qquad \text{in}\,(0,\infty)\times\Omega, \tag{1.2}$$

subject to appropriate initial and boundary data, is well known for us, here u represents the particle density and v is the density of the external signal, χ is the sensitive coefficient, Ω is either a bounded domain in R^N with smooth boundary or the whole space R^N, and the time constant $0\le\tau\le 1$ indicates that the spatial spread of the organisms u and the control signal v are on different time scales. The case $\tau=0$ corresponds to a quasi-steady state assumption for the signal distribution [1,2,3].

The above parabolic models are based on the ensemble average movement of populations as a whole. To compare with these models, Hillen and Stevens [4], based on the individual movement properties of the species, introduced a 1-dimensional hyperbolic chemotaxis model to describe the response of the individuals to an external chemical or its gradient. They studied the existence and blowup of the weak solution in Sobolev

*Corresponding author. *Email addresses:* `kaizhoucm@whu.edu.cn` (K. Zhou), `wush8@sina.com` (S. H. Wu)

spaces (here blowup means that the Sobolev norm of the solution will be unbounded). Furthermore Hillen and Levine [5] constructed even a special blow-up solution in the case of 1-dimensional hyperbolic chemotaxis model in which there is no diffusion for the external stimulus.

When the external stimulus is based on the light (or the electromagnetic wave), Chen and Wu [6] introduced following system,

$$u_t=\nabla(\nabla u-\chi(v)\nabla v\cdot u), \qquad \text{in}\,(0,\infty)\times\Omega, \tag{1.3}$$

$$v_{tt}=\Delta v+g(v,u), \qquad \text{in}\,(0,\infty)\times\Omega, \tag{1.4}$$

where, for example, if the external signal is the electromagnetic field, then v would be voltage (in this case ∇v denotes the electromagnetic field).

Specificly, for the system (3)-(4) with following initial and boundary conditions,

$$\begin{cases} \dfrac{\partial u}{\partial n}=\dfrac{\partial v}{\partial n}=0, & \text{on } (0,T)\times\partial\Omega, \\ u(0,\cdot)=u_0, \quad v(0,\cdot)=\varphi, \quad v_t(0,\cdot)=\psi \text{ in } \Omega, & \end{cases} \tag{1.5}$$

they studied the case of $g(v,u)=-\gamma v+f(u)$ in [6] and the case of $g(v,u)=\alpha uv$ and $g(v,u)=h(v^2)v+f(u)$ in [7] respectively, where $\Omega\subset\mathbf{R}^N$, a bounded open domain with smooth boundary $\partial\Omega$ and n is the unit outer normal on $\partial\Omega$.

Wu et al.[8,9] studied the case when the domain is a compact Riemanian Manifold M of n-D without boundary. Chen et al.[10] studied the following system,

$$\begin{cases} u_t=\nabla[\nabla u-u\nabla(v+\ln W)], & \text{in}\,(0,T)\times\Omega, \\ v_{tt}-\Delta_N v+av=u, & \text{in}\,(0,T)\times\Omega. \end{cases} \tag{1.6}$$

In this article, we discuss the problem (1.3)-(1.4) with $\Omega=R^N$ and obtain the local and global existence of the weak solutions for $1\le N\le 3$ and $N=1$ respectively.

2 Main results

Consider

$$\begin{cases} u_t=\nabla(\nabla u-\chi u\nabla v), & \text{in } (0,T)\times\mathbf{R}^N, \\ v_{tt}=\Delta v+g(u,v), & \text{in } (0,T)\times\mathbf{R}^N, \\ u(0,\cdot)=u_0, & \text{in } \mathbf{R}^N, \\ v(0,\cdot)=\varphi,\, v_t(0,\cdot)=\psi, & \text{in } \mathbf{R}^N, \end{cases} \tag{2.1}$$

where χ is a nonnegative constant.

Choose a constant σ, which satisfies

$$1<\sigma<2, \tag{2.2}$$