JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **29**, No. 2, pp. 102-115

doi: 10.4208/jpde.v29.n2.2
June 2016

A Multiplicity Result for Some Integro-Differential Biharmonic Equation in $\mathbb{R}^4$

AOUAOUI Sami*

Institut Supérieur des Mathématiques Appliquées et de l'Informatique de Kairouan
Avenue Assad Iben Fourat, 3100 Kairouan, Tunisie.

Received 22 December 2015; Accepted 3 June 2016

Abstract. In this paper, we prove the existence of at least two nontrivial solutions for some biharmonic elliptic equation involving an integral term. The nonlinear term exhibits an exponential growth at infinity. Our method consists of a combination between variational tools and iterative technique.

AMS Subject Classifications: 35A25, 35D30, 35J35, 35J62, 35J70

Chinese Library Classifications: O175.25, O175.4

Key Words: Integro-differential; exponential growth; iterative scheme; multiplicity; radial solution; Adams inequality; principle of symmetric criticality.

1 Introduction and statement of main results

In this paper, we investigate the existence of multiple solutions to the equation

$$(P)\quad \Delta^2 u+u=f(x,u)+\int_{\mathbb{R}^4}k(x,y)g(u(y),\Delta u(y))dy+h(x),\ x\in\mathbb{R}^4,$$

where f,k,g and h are some functions satisfying the following assumptions
(H_1) $f:\mathbb{R}^4\times\mathbb{R}\to\mathbb{R}$ is a continuous function which is spherically symmetric with respect to $x\in\mathbb{R}^4$, that is

$$\forall\ (x,y,s)\in\mathbb{R}^4\times\mathbb{R}^4\times\mathbb{R},\ \ |x|=|y|\Rightarrow f(x,s)=f(y,s).$$

Moreover, we assume that there exist $\alpha>1$, $\beta>1$, $p>0$, $C_0>0$ and $C_1>0$ such that

$$|f(x,s)|\le C_0\left(|s|^{\alpha}+|s|^{\beta}\left(e^{ps^2}-1\right)\right),\ \forall\ x\in\mathbb{R}^4,\ \forall\ s\in\mathbb{R},$$

*Corresponding author. *Email address:* `sami_aouaoui@yahoo.fr` (S. Aouaoui)

and

$$F(x,s)=\int_0^s f(x,t)dt \geq C_1|s|^{\alpha+1},\ \forall\ x\in\mathbb{R}^4,\ \forall\ s\in\mathbb{R}.$$

(H_2) There exists $\nu>2$ such that

$$0<\nu F(x,s)\leq f(x,s)s,\ \forall\ x\in\mathbb{R}^4,\ \forall\ s\in\mathbb{R}\backslash\{0\}.$$

(H_3) For any $z\geq 0$, there exists a positive constant L_z such that

$$\left|f(x,s)-f(x,s')\right|\leq L_z\left|s-s'\right|,\ \forall\ x\in\mathbb{R}^4,\ \forall\ (s,s')\in[-z,z]^2.$$

(H_4) $k:\mathbb{R}^4\times\mathbb{R}^4\to[0,+\infty[$ is some function lying in $L^2(\mathbb{R}^4\times\mathbb{R}^4)$ such that $k\neq 0$ and k is spherically symmetric (radial) with respect to x, that is

$$\forall\ (x_1,x_2,y)\in\mathbb{R}^4\times\mathbb{R}^4\times\mathbb{R}^4,\ \ |x_1|=|x_2|\Rightarrow k(x_1,y)=k(x_2,y).$$

(H_5) $g:\mathbb{R}\times\mathbb{R}\to[0,+\infty[$ is some continuous function such that $g(0,0)=0$. Moreover, we assume that there exists a constant $L_0>0$ such that

$$\left|g(a,b)-g(a',b')\right|\leq L_0\left(\left|a-a'\right|+\left|b-b'\right|\right),\ \forall\ (a,b,a',b')\in\mathbb{R}^4.$$

(H_6) $h:\mathbb{R}^4\to[0,+\infty[$ is spherically symmetric (radial) such that $h\in L^2(\mathbb{R}^4)$.

Examples: A typical example of f verifying $(H_1)-(H_3)$ is given by

$$f(x,s)=f(s)=Ce^{ps^2}\left(q|s|^{q-2}s+2ps|s|^q\right),$$

where $C>0$, $p>0$ and $q>2$. Clearly, $F(x,s)=C|s|^q e^{ps^2}$. It suffices to note that (H_2) holds with $\nu=q>2$ and that (H_3) holds because the derivative f' exists and is continuous. Concerning the function g, one can take

$$g(a,b)=g_1(a)+g_2(b),\ (a,b)\in\mathbb{R}^2,$$

where g_1 and g_2 are two Lipschitz continuous and nonnegative functions in $\mathbb{R}$ (example: $g_1(a)=g_2(a)=|\sin a|,\ \forall\ a\in\mathbb{R}$). Finally, for k, one can choose the function

$$k(x,y)=\frac{1}{\left(1+|x|^3\right)\left(1+|y|^3\right)},\ \forall\ (x,y)\in\mathbb{R}^4\times\mathbb{R}^4.$$

In the last decades, there has been a wide literature of higher order elliptic equations and especially those involving a biharmonic operator (i.e of fourth order). This big interest can be motivated by their role in modeling various physical phenomena such as