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Asymptotic Behavior of the Solution to a 3-D Simplified Energy-Transport Model for Semiconductors

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Abstract. The well-posedness of smooth solution to a 3-D simplified Energy-Transport model is discussed in this paper. We prove the local existence, uniqueness, and asymptotic behavior of solution to the equations with hybrid cross-diffusion. The smooth solution convergences to a stationary solution with an exponential rate as time tends to infinity when the initial date is a small perturbation of the stationary solution.

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1 Introduction

Energy-Transport model was first proposed by Stratton [1] and latter derived from the semiconductor Boltzmann equation by Ben Abdallah et al. [2]. The strong coupling and temperature gradients make it difficult to analyze the energy-transport model. Therefore, we consider in this paper a simplified energy-transport model which still includes temperature gradients with weakly coupling of the energy equation.

The simplified Energy-Transport model, achieved by Jüngel et al. in [3], consists of a drift-diffusion-type equation for the electron density $n(x,t)$, a nonlinear heat equation for the electron temperature $\theta(x,t)$, and the Poisson equation for the electric potential $V(x,t)$:

$$\partial_t n - \mathrm{div}(\nabla(n\theta) - n\nabla V) = 0, \tag{1.1}$$

$$\mathrm{div}(\kappa(n)\nabla\theta) = \frac{n}{\tau}(\theta - \theta_L(x)), \tag{1.2}$$

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$$\lambda^2 \triangle V = n - C(x). \tag{1.3}$$

Here, $\kappa(n)$ is the thermal conductivity, we suppose that $\kappa(n)=n$, $\theta_L(x)$ is the lattice temperature, and $C(x)$ is the doping profile characterizing the device under consideration. The energy relaxation time $\tau > 0$ and the Debye length $\lambda > 0$ are scaled physical parameters. Without lose of generality, we suppose that $\tau = \theta_L(x) = \lambda = 1$, and set $E(x,t) = \nabla V(x,t)$. Then the model (1.1)-(1.3) can be changed into the following model for the electron density $n(x,t)$, the electron temperature $\theta(x,t)$ and the electric field $E(x,t)$:

$$\partial_t n - \mathrm{div} j = 0, \quad j = (\nabla(n\theta) - nE), \tag{1.4}$$
$$\mathrm{div}(n\nabla\theta) = n(\theta - 1), \tag{1.5}$$
$$\mathrm{div} E = n - C(x). \tag{1.6}$$

Eqs. (1.4)-(1.6) hold in the bounded main $\Omega \subset R^3$, with the initial boundary condition

$$n(x,0) = n_0(x), \tag{1.7}$$
$$j \cdot \vec{n}|_{\partial\Omega} = 0, \quad \nabla\theta \cdot \vec{n}|_{\partial\Omega} = 0, \quad E \cdot \vec{n}|_{\partial\Omega} = 0, \tag{1.8}$$

where $\vec{n}$ denotes the exterior unit normal vector on $\partial\Omega$, and the initial datum $n_0(x)$ satisfies the following condition

$$\int_\Omega n_0(x) - C(x) \mathrm{d}x = 0. \tag{1.9}$$

Before we exposit our results, we review the energy-transport model in the literature. The common form for energy-transport model [4] is

$$\begin{aligned}
&\partial_t n - \frac{1}{q} \mathrm{div} J_n = 0,\\
&\partial_t U(n,\theta) - \mathrm{div} J_w = -J_n \cdot \nabla V + W(n,\theta),\\
&\lambda^2 \triangle V = n - C(x),
\end{aligned}$$

with

$$\begin{aligned}
J_n &= L_{11}\left(\frac{\nabla n}{n} - \frac{q\nabla V}{k_B\theta}\right) + \left(\frac{L_{12}}{k_B\theta} - \frac{3}{2}L_{11}\right)\frac{\nabla\theta}{\theta},\\
qJ_w &= L_{21}\left(\frac{\nabla n}{n} - \frac{q\nabla V}{k_B\theta}\right) + \left(\frac{L_{22}}{k_B\theta} - \frac{3}{2}L_{21}\right)\frac{\nabla\theta}{\theta},
\end{aligned}$$

where $U(x,\theta)$ is the density of the internal energy, $W(n,\theta)$ is the energy relaxation term satisfying $W(n,\theta)(\theta - \theta_L(x)) \le 0$,

$$W(n,\theta) = -\frac{n(\theta - \theta_L(x))}{\tau_\beta}, \quad \tau_\beta = \frac{\pi^{\frac{5}{2}}\theta^{\frac{1}{2}-\beta}}{\sqrt{8}\Gamma(\beta+2)s_0},$$