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Exponential Decay of Energy for a Logarithmic Wave Equation

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Abstract. In this paper we consider the initial boundary value problem for a class of logarithmic wave equation. By constructing an appropriate Lyapunov function, we obtain the decay estimates of energy for the logarithmic wave equation with linear damping and some suitable initial data. The results extend the early results.

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Key Words: Logarithmic wave equation; initial boundary value problem; decay estimate.

1 Introduction

In this paper, we shall deal with decay estimates of energy for the initial boundary value problem of the logarithmic wave equation

$$u_{tt}-\Delta u+u+u_t+|u|^2u=u\ln|u|^k, \qquad x\in\Omega,\ t>0, \tag{1.1}$$

$$u(x,t)=0, \qquad x\in\partial\Omega,\ t>0, \tag{1.2}$$

$$u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \qquad x\in\Omega, \tag{1.3}$$

where $\Omega\subset R^n, n\geq 1$, is a bounded domain with smooth boundary $\partial\Omega$, $k\geq 1$.

This type of problems have many applications in many branches of physics such as nuclear physics, optics and geophysics (see [1–3] and their references). It has been also introduced in the quantum field theory, such kinds nonlinearity appear naturally in inflation cosmology and in supersymmetric field theories([3]). When Ω is a finite interval

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$[a,b]$ in problem (1.1)-(1.3) without damping term u_t, it is a relativistic version of logarithmic quantum mechanics introduced by Bialynicki-Birula and Mycielski (see [1, 4]). Hiramatsu et.al [5] introduced also the following equation

$$u_{tt}-\Delta u+u+u_t+|u|^2u=u\ln|u|, \qquad x\in\Omega,\ t>0, \tag{1.4}$$

for studying the dynamics of Q-ball in theoretical physics.

In [6], Cazenave and Haraux established the existence and uniqueness of a solution for the Cauchy problem for the following equation

$$u_{tt}-\Delta u=u\ln|u|^k, \tag{1.5}$$

in R^3. By using compactness method Gorka [3] obtained the global existence of weak solutions for all $u_0\in H_0^1, u_1\in L^2$ to the initial boundary value problem of Eq. (1.5) in one-dimensional case. In [2], Bartkowski and Gorka showed the existence of classical solutions and investigated weak solutions for the corresponding Cauchy problem for Eq. (1.5) in one-dimensional case. In [7], Han got the global existence of weak solutions for all $u_0\in H_0^1, u_1\in L^2$ to the initial boundary value problem (1.4) in R^3. However, no result is given concerning the decay property of solutions and it is desirable to establish the uniform stabilization of solutions to (1.1)-(1.3).

We should also point out that there is extensive literature on the question of existence, asymptotic behavior and nonexistence of solution for the following initial boundary value problem

$$u_{tt}-\Delta u+h(u_t)=f(u), \qquad x\in\Omega,\ t>0, \tag{1.6}$$
$$u(x,t)=0, \qquad x\in\partial\Omega,\ t>0, \tag{1.7}$$
$$u(x,0)=u_0(x), \quad u_t(x,0)=u_1(x), \qquad x\in\Omega. \tag{1.8}$$

Here, we mention only some results about the interaction between the damping term and the source term. It was first considered by Levine [8] in the linear damping case $h(u_t)=au_t$ and a polynomial source term of the form $f(u)=b|u|^{p-2}u$. Then Georgiev and Todorova in [9] extended Levine's result to the nonlinear damping case $h(u_t)=a|u_t|^{m-2}u_t$. For the exponential decay for the initial boundary value problem for the following equation

$$u_{tt}-\Delta u-\omega\Delta u_t+\nu u_t=u|u|^{p-2}, \tag{1.9}$$

has been studied for $\omega\geq 0$ or $\nu\geq 0$, among others, by Nakao [10], Zuazua [11, 12] Benaissa [13] and Gebri [14]. However, the method in above mentioned works cannot be applied directly to the case that the equations have the logarithmic nonlinearities term. Introducing the logarithmic nonlinearity term makes the problem different from the one considered in above all mentioned paper. The purpose of this paper is to obtain a decay estimate of solutions to the problem (1.1)-(1.3). More precisely we show that we can find