JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **27**, No. 2, pp. 136-142

doi: 10.4208/jpde.v27.n2.4
June 2014

The Tanh Method for Kink Solution of Some Modified Nonlinear Equation

FOROZANI Gh.* and GHORVEEI NOSRAT M.

Department of Physics, Faculty of Science, Bu-Ali Sina University, Hamedan, Iran.

Received 22 April 2013; Accepted 28 February 2014

Abstract. The tanh method is a very powerful technique for computation of exact traveling wave, in this paper this method has been employed for special modified states of Burger, Klein-Gordon and Fisher-Burger equations and the solitary solution of these equations are derived.

AMS Subject Classifications: 76B25, 37K40, 34A34, 34G20

Chinese Library Classifications: O175.14

Key Words: Tanh method; nonlinear equation; solitary solution.

1 Introduction

The nonlinear phenomena are very important in a variety of scientific fields, especially in fluid mechanics, solid state physics, plasma physics, plasma waves, nonlinear optics and etc [1]. A variety of powerful method such as the inverse scattering [2], the Backlund transformation [3,4], sine-cos method [5], tanh-sech method [6], Hirota's bilinear technique [7] and the homogeneous balance method [8] were used to solve nonlinear equations. The tanh method developed by Malfliet et al. [9,10], is a reliable and accurate algebraic method to obtain exact solution of nonlinear equations [11]. Wazwaz also used this method for several forms of nonlinear partial differential equations such as

Fisher-Burger equation:

$$u_t+(n+1)u^n u_x+u_{xxx}=0; \tag{1.1}$$

Klein-Gordon equation:

$$u_t t-a^2 u_x x+(\alpha)u-(\beta)u^n=0; \tag{1.2}$$

*Corresponding author. *Email addresses:* g.forozani@gmail.com (Gh. Forozani), m.ghorveei@yahoo.com (M. Ghorveei Nosrat)

Burger equation:

$$u_t + uu_x + u_x x = 0. \tag{1.3}$$

Fisher-Burger equation has important applications in various fields such as traffic flow, financial mathematics, gas dynamic, applied mathematics and physics [12-19]. This equation shows a prototypical model for describing the interaction between the reaction mechanism, convection effect, and diffusion transport [20]. In this paper solitary solution of theirs special modified states will be obtained.

2 Tanh method

A partial differential equation (PDE) can be converted to an ordinary differential equation (ODE) upon using a wave variable [21]:

$$z = x - ct. \tag{2.1}$$

Introducing a new independent variable $y = \tanh \mu z$ that leads to change of derivatives[7]:

$$\frac{\mathrm{d}}{\mathrm{d}z} = (1-y^2)\frac{\mathrm{d}}{\mathrm{d}y}, \tag{2.2}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2} = (1-y^2) - 2y\frac{\mathrm{d}}{\mathrm{d}y} + (1-y^2)\frac{\mathrm{d}^2}{\mathrm{d}y^2}, \tag{2.3}$$

$$\frac{\mathrm{d}^3}{\mathrm{d}z^3} = (1-y^2)[(6y^2-2)]\frac{\mathrm{d}}{\mathrm{d}y} - 6y(1-y^2)\frac{\mathrm{d}^2}{\mathrm{d}y^2} + (1-y^2)^2\frac{\mathrm{d}^3}{\mathrm{d}z^3}, \tag{2.4}$$

Introduce the ansatz:

$$u(\mu z) = S(y) = \sum_{k=0}^{m} a_k y^k, \tag{2.5}$$

where m in most cases is a positive integer. To determine the parameter m we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. With m determined, equate the coefficients of power of y in the resulting equation [1].

3 Modified nonlinear Burgers equation

By adding a nonlinear term of the form $u(1-u^2)$ to burger's equation, the modified burger equation is obtained as follows:

$$u_t + uu_x - u_{xx} + u1 - u^2 = 0. \tag{3.1}$$

Substituting (2.1) into (3.1) gives:

$$cu' + uu' - u'' + u - u^3 = 0. \tag{3.2}$$