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Neutral Functional Partial Differential Equations Driven by Fractional Brownian Motion with Non-Lipschitz Coefficients

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Abstract. Under a non-Lipschitz condition being considered as a generalized case of Lipschitz condition, the existence and uniqueness of mild solutions to neutral stochastic functional differential equations driven by fractional Brownian motion with Hurst parameter $1/2 < H < 1$ are investigated. Some known results are generalized and improved.

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1 Introduction

Recently, the theory for stochastic differential equations (without delay) driven by a fractional Brownian motion (fBm) has been studied intensively (see e.g. [1–6] and the references therein).

As for the stochastic functional differential equations driven by a fBm, even much less has been done, as far as we know, there exists only a few papers published in this field. In [7], the authors studied the existence and regularity of the density by using the Skorohod integral based on Malliavin calculus. In [8], Neuenkirch et al. studied the problem by using rough path analysis. In [9], Ferrante and Rovira studied the existence and convergence when the delay goes to zero by using the Riemann-Stieltjes integral. Using

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also the Riemann-Stieltjes integral, [10] proved the existence and uniqueness of mild solution in infinite dimensional space. In infinite dimensional space, [11] have discussed the existence, uniqueness and exponential asymptotic behavior of mild solutions by using Wiener integral. Very recently, [12] first investigated the following neutral stochastic functional differential equations driven by a fractional Brownian motion under the global Lipschitz and linear growth condition

$$\begin{cases} \mathrm{d}[x(t)+g(t,x(t-r(t)))]=[Ax(t)+f(t,x(t-\rho(t)))]\mathrm{d}t+\sigma(t)\mathrm{d}B^H(t), \quad 0\le t\le T, \\ x(t)=\varphi(t), \quad t\in[-\tau,0]. \end{cases} \tag{1.1}$$

Where A is the infinitesimal generator of an analytic semigroup of bounded linear operators, $(S(t))_{t\ge0}$, in a Hilbert space X, B^H is a Q-fractional Brownian motion on a real and separable Hilbert space Y, $r,\rho:[0,T]\to[0,\tau]$ $(\tau>0)$ are continuous, $f,g:[0,T]\times X\to X$, $\sigma:[0,T]\to\mathcal{L}_2^0(Y,X)$ are appropriate functions and $\varphi\in C([-\tau,0];L^2(\Omega,X))$. Here $\mathcal{L}_2^0(Y,X)$ denotes the space of all Q-Hilbert-Schmidt operators from Y into X (see Section 2).

Unfortunately, for many practical situations, the nonlinear terms do not obey the global Lipschitz and linear growth condition, even the local Lipschitz condition. Motivated by the above papers, in this paper, we aim to extend the existence and uniqueness of mild solutions to cover a class of more general neutral stochastic functional differential equations driven by a fractional Brownian motion with Hurst parameter $1/2<H<1$ under a non-Lipschitz condition, with the Lipschitz condition being regarded as a special case, and a weakened linear growth condition.

The rest of this paper is organized as follows. In Section 2, we introduce some notations, concepts, and basic results about fractional Brownian motion, Wiener integral over Hilbert spaces and we recall some preliminary results about analytic semigroups and fractional power associated to its generator. In Section 3, the existence and uniqueness of mild solutions are proved.

2 Preliminaries

In this section we collect some notions, conceptions and lemmas on Wiener integrals with respect to an infinite dimensional fractional Brownian motion. In addition, we also recall some basic results about analytical semi-groups and fractional powers of their infinitesimal generators which will be used throughout the whole of this paper.

Let $(\Omega,\mathcal{F},\mathbb{P})$ be a complete probability space. Consider a time interval $[0,T]$ with arbitrary fixed horizon T and let $\{\beta^H(t),\ t\in[0,T]\}$ be the one-dimensional fractional Brownian motion with Hurst parameter $H\in(1/2,1)$. This means by definition that β^H is a centered Gaussian process with covariance function:

$$R_H(t,s)=\mathbb{E}(\beta_t^H\beta_s^H)=\frac{1}{2}(t^{2H}+s^{2H}-|t-s|^{2H}).$$