JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **26**, No. 2, pp. 99-106

doi: 10.4208/jpde.v26.n2.1
June 2013

A Remark on the Existence of Positive Solution for a Class of (p,q)-Laplacian Nonlinear System with Multiple Parameters and Sign-Changing Weight

RASOULI S. H.*

Department of Mathematics, Faculty of Basic Sciences, Babol University of Technology, Babol, Iran.

Received 14 February 2012; Accepted 21 February 2013

Abstract. The paper deal with the existence of positive solution for the following (p,q)-Laplacian nonlinear system

$$\begin{cases} -\Delta_p u = a(x)\big(\alpha_1 f(v)+\beta_1 h(u)\big), & x\in\Omega, \\ -\Delta_q v = b(x)\big(\alpha_2 g(u)+\beta_2 k(v)\big), & x\in\Omega, \\ u=v=0, & x\in\partial\Omega, \end{cases}$$

where Δ_p denotes the p-Laplacian operator defined by

$$\Delta_p z = \mathrm{div}(|\nabla z|^{p-2}\nabla z),$$

$p>1$, α_1, α_2, β_1, β_2 are positive parameters and Ω is a bounded domain in $R^N(N>1)$ with smooth boundary $\partial\Omega$. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and f, g, h, k are C^1 nondecreasing functions such that $f,g,h,k\colon[0,\infty)\to[0,\infty)$; $f(s)$, $g(s)$, $h(s)$, $k(s)>0$; $s>0$ and

$$\lim_{x\to\infty}\frac{f\big(Mg(x)^{\frac{1}{q-1}}\big)}{x^{p-1}}=0$$

for every $M>0$.
We discuss the existence of positive solution when f, g, h, k, $a(x)$ and $b(x)$ satisfy certain additional conditions. We use the method of sub-super solutions to establish our results.

AMS Subject Classifications: 35J55, 35J65

Chinese Library Classifications: O175.25, O175.8

Key Words: (p,q)-Laplacian nonlinear system; multiple parameters; sign-changing weight.

*Corresponding author. *Email address:* s.h.rasouli@nit.ac.ir (S. H. Rasouli)

1 Introduction

This study concerns the existence of positive solution for the nonlinear system

$$\begin{cases} -\Delta_p u = a(x)\big(\alpha_1 f(v)+\beta_1 h(u)\big), & x\in\Omega, \\ -\Delta_q v = b(x)\big(\alpha_2 g(u)+\beta_2 k(v)\big), & x\in\Omega, \\ u=v=0, & x\in\partial\Omega, \end{cases} \tag{1.1}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = \mathrm{div}(|\nabla z|^{p-2}\nabla z)$, $p>1$, α_1, α_2, β_1, β_2 are positive parameters and Ω is a bounded domain in $R^N (N>1)$ with smooth boundary $\partial\Omega$. Here $a(x)$ and $b(x)$ are C^1 sign-changing functions that maybe negative near the boundary and $f,g,h,k:[0,\infty)\to[0,\infty)$ are C^1 nondecreasing functions such that $f(s)$, $f(g)$, $h(s)$, $k(s)>0$ for $s>0$.

Problems involving the p-Laplace operator arise in some physical models like the flow of non-Newtonian fluids: pseudo-plastic fluids correspond to $p\in(1,2)$ while dilatant fluids correspond to $p>2$. The case $p=2$ expresses Newtonian fluids [1]. On the other hand, quasilinear elliptic systems like (1.1) has an extensive practical background. It can be used to describe the multiplicate chemical reaction catalyzed by the catalyst grains under constant or variant temperature, it can be used in the theory of quasiregular and quasiconformal mappings in Riemannian manifolds with boundary (see [2,3]) and can be a simple model of tubular chemical reaction, more naturally, it can be a correspondence of the stable station of dynamical system determined by the reaction-diffusion system, see Ladde and Lakshmikantham et al. [4]. More naturally, it can be the populations of two competing species [5]. So, the study of positive solutions of elliptic systems has more practical meanings.We refer to [6–9] for additional results on elliptic problems involving the p-Laplacian.

For the single-equation, namely equation of the form

$$\begin{cases} -\Delta_p u = \lambda a(x) f(u), & x\in\Omega, \\ u=0, & x\in\partial\Omega, \end{cases}$$

with sign-changing weight function has been studied by several authors (see [10, 11]) and [12] for $p=2$. See [13,14] where the authors discussed the system (1.1) when $p=q=2$, $a\equiv1$, $b\equiv1$, $\alpha_1=\alpha_2$, $\beta_1=\beta_2=0$, f, g are increasing and $f,g\geq0$. In [15], the authors extended the study of [13], to the case when no sign conditions on $f(0)$ or $g(0)$ were required and in [16] they extend this study to the case when $p=q>1$. In a recent paper [17], the authors studied the system (1.1) when $a\equiv1$, $b\equiv1$. Here we focus on further extending the study in [17] to the system (1.1). In fact, we focus on sign-changing weight functions $a(x)$ and $b(x)$. Due to this weights functions, the extensions are challenging and nontrivial. Our approach is based on the method of sub-super solutions, see [18, 19]. Several methods have been used to treat quasilinear equations and systems. In the scalar case, weak solutions can be obtained through variational methods which provide critical points of