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ON THE LOCAL WELL-POSEDNESS FOR THE DUMBBELL MODEL OF POLYMERIC FLUIDS*

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Abstract In this paper, we consider the local well-posedness of smooth solutions to a coupled microscopic-macroscopic model for polymeric fluid in general space dimension and with general initial data in Hölder space.

Key Words Polymeric fluids; Navier-Stokes equations; Littlewood-Paley decomsposition.

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1. Introduction

In this paper, we study the local well-posedness of smooth solutions to a coupled microscopic-macroscopic model for polymeric fluid in general space dimension and with general initial data in Hölder space. The micro-mechanical models for polymeric liquids usually consist of beads joined by springs or rods [1,2]. In the simplest case, a molecule configuration can be described by its end-to-end vector Q. Taking into account the elastic effect together with the thermo-fluctuation, the distribution function $\psi(t,x,Q)$ of molecule orientations Q satisfies a Fokker-Planck equation. The convection velocity u satisfies the Navier-Stokes equations with an elastic stress which reflects the microscopic contribution of the polymer molecules to the overall macroscopic flow fields. Mathematically, this system reads (one may check [3] for a formal energetic variational

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derivation)

$$\begin{cases} u_t + u\cdot\nabla u + \nabla p = \triangle u + \mathtt{div}\,\tau, \qquad x\in\mathbf{R}^d,\\ \mathtt{div}\,u = 0, \quad x\in\mathbf{R}^d,\\ \psi_t + u\cdot\nabla\psi = \triangle_Q\psi - \nabla_Q\cdot(\nabla u Q\psi - \nabla_Q U\psi), \quad (x,Q)\in\mathbf{R}^d\times\mathbf{R}^d, \end{cases} \tag{1.1}$$

with the polymer stress tensor τ being given by

$$\tau = \int_{\mathbf{R}^d}\nabla_Q U\otimes Q\psi\mathrm{d}Q, \tag{1.2}$$

and $U(Q) = U(|Q|^2)$ being the potential function.

Existence results for micro-macro models of polymeric fluids are usually limited to small time existence and uniqueness of strong solutions [4] or global existence of weak solution [5]. In particular, the authors [6] proved the local wellposedness of dumbbell model of polymeric fluid with initial data in weighted Sobolev space. In the setting when the last equation of (1.1) is formulated as a stochastic PDE, we refer to [7] (see also [8] for a polynomial force). With regards to PDE coupled system, we refer the reader to the earlier work [9].

In the work [3], the authors proved the global existence of smooth solutions to (2.5) near equilibrium, which is a sort of extension to a related result of the Oldroyd model [10], and which corresponds to the Hooke dumbbell model. In two space dimensions, the authors [11] proved the global existence of smooth solutions to a coupled nonlinear Fokker-Planck and Navier-Stokes system when the convection velocity u in the Fokker-Planck equation is replaced by a sort of time averaged one. Later this assumptions was moved by Constantin and Masmoudi [12]. At the same time, Lin, Zhang and Zhang [13] independently proved the global regularity for the 2D co-rotational FENE model. The main ideas in [12, 13] are the so-called losing derivative estimate from [14, 15]. More recently, the authors [16] extended the result in [13] to the case of co-rational dumbbell model, furthermore, by using the argument in [13], they even obtained the time growth estimate of the global solutions in Sobolev norm

Our new observation in this paper is that we do not need any decay assumption in the x variables in order to prove the local wellposedness of (1.1). Instead we only need to take the initial data in sort of Hölder space.

In the following, we always assume the potential function $U(Q)$ satisfies either the following Assumptions **A** or **B**:

Assumption A

$$|Q|\le C(1+|\nabla_Q U|), \qquad |\nabla_Q U|\le C|Q|, \qquad |\triangle_Q U|\le C. \tag{1.3}$$

Assumption B There exits some $a>1$ such for integers $0\le k\le 2$

$$\begin{aligned} &|Q|^a\le C(1+|\nabla_Q U|), \quad \triangle_Q U\le C+\delta|\nabla_Q U|^2 \text{ with } \delta<1,\\ &\int_{\mathbf{R}^d}|\nabla_Q U|^2 e^{-U}\mathrm{d}Q\le C, \quad \int_{\mathbf{R}^d}|Q|^4 e^{-U}\mathrm{d}Q\le C, \end{aligned} \tag{1.4}$$