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RESOLVING THE SINGULARITIES OF THE MINIMAL HOPF CONES*

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Dedicate to Professor K. C. Chang on his seventieth birthday

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Abstract We resolve the singularities of the minimal Hopf cones by families of regular minimal graphs.

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1. Introduction

In this paper, we resolve the singularities of the minimal Hopf cones found in Lawson and Osserman [1]. The Lipschitz yet non C^1 minimal graph cone in $\mathbb{R}^{2m} \times \mathbb{R}^{m+1}$ is

$$C_m = \left\{ \left(x, S_m \frac{H(x)}{r} \right) : x \in \mathbb{R}^{2m} \right\},$$

where $m = 2, 4, 8,$ $S_m = \sqrt{\frac{2m+1}{4(m-1)}}$, $r = |x|$, and the Hopf map $H : \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^{m+1}$ is defined as follows. One identifies $\mathbb{R}^m$ with the normed algebra, complex numbers $\mathbb{C}$ $(m = 2)$, quaternions $\mathbb{H}$ $(m = 4)$, and octonions $\mathbb{O}$ $(m = 8)$. Let $x = (u, v) \in \mathbb{R}^m \times \mathbb{R}^m$, then

$$H(x) = \left(|u|^2 - |v|^2, 2v\bar{u} \right).$$

For each of the minimal Hopf cones, we prove there exist a family of regular minimal graphs in $\mathbb{R}^{2m} \times \mathbb{R}^{m+1}$ whose tangent cone at ∞ are the minimal Hopf cone C_m. To be precise, we have

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Theorem 1.1 *There exist a family of analytic minimal graphs*

$$G_\mu = \left\{ \left(x, \mu^{-1} f(\mu r) \frac{H(x)}{r^2} \right) : \quad x \in \mathbb{R}^{2m} \right\}$$

for $m = 2,\ 4,\ 8,$ *where* $\mu > 0$ *and* f *satisfies*

$$0 \le f(r) < S_m r,$$
$$0 \le f_r(r);$$

and for small r *near* 0

$$f(r) = O\left(r^2\right),$$
$$f_r(r) = O(r);$$

while for large r

$$f(r) = S_m r + O\left(\frac{1}{r^\delta}\right),$$
$$f_r(r) = S_m + O\left(\frac{1}{r^{1+\delta}}\right)$$

with $\delta = m - \sqrt{m^2 - 2m + \frac{1}{2m}} - 1 > 0.$

Further we have another family of minimal graphs which are "above" each of the minimal Hopf cones in the sense that $f(r) > S_m r$. Their tangent cones at ∞ are still the minimal Hopf cone C_m. This family of minimal graphs are only regular away from $0 \times \mathbb{R}^{m+1}$, but have finite area near the singular points.

Theorem 1.1. Theorem 1.2 *There exist a family of analytic minimal graphs*

$$G_\mu = \left\{ \left(x, \mu^{-1} f(\mu r) \frac{H(x)}{r^2} \right) : \quad x \in \mathbb{R}^{2m} \backslash \{0\}, \right\}$$

for $m = 2,\ 4,\ 8,$ *where* $\mu > 0$ *and* f *satisfies*

$$f(r) > S_m r,$$
$$f_r(r) \ge 0;$$

for small r *near* 0

$$f(r) = O(1),$$
$$f_r(r) = O(r);$$

for large r

$$f(r) = S_m r + O\left(\frac{1}{r^\delta}\right),$$
$$f_r(r) = S_m + O\left(\frac{1}{r^{1+\delta}}\right).$$

Moreover, in the case $m = 2,$ *one can take* $\delta = m + \sqrt{m^2 - 2m + \frac{1}{2m}} - 1 = \frac{3}{2}.$