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OPERATORS ON CORNER MANIFOLDS WITH EXIT TO INFINITY

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Abstract We study (pseudo-)differential operators on a manifold with edge Z, locally modelled on a wedge with model cone that has itself a base manifold W with smooth edge Y. The typical operators A are corner degenerate in a specific way. They are described (modulo 'lower order terms') by a principal symbolic hierarchy $\sigma(A) = (\sigma_\psi(A), \sigma_\wedge(A), \sigma_{\boldsymbol{\wedge}}(A))$, where σ_ψ is the interior symbol and $\sigma_\wedge(A)(y,\eta)$, $(y,\eta) \in T^*Y \setminus 0$, the (operator-valued) edge symbol of 'first generation', cf. [1]. The novelty here is the edge symbol $\sigma_{\boldsymbol{\wedge}}$ of 'second generation', parametrised by $(z,\zeta) \in T^*Z \setminus 0$, acting on weighted Sobolev spaces on the infinite cone with base W. Since such a cone has edges with exit to infinity, the calculus has the problem to understand the behaviour of operators on a manifold of that kind.

We show the continuity of corner-degenerate operators in weighted edge Sobolev spaces, and we investigate the ellipticity of edge symbols of second generation. Starting from parameter-dependent elliptic families of edge operators of first generation, we obtain the Fredholm property of higher edge symbols on the corresponding singular infinite model cone.

Key Words Operators on manifolds with edge and conical exit to infinity; Sobolev spaces with double weights on singular cones; parameter-dependent ellipticity; edge and corner symbols.

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1. Introduction

This paper studies (pseudo-)differential operators on manifolds with conical exit to infinity whose cross section is a (compact) manifold W with smooth edge Y. More precisely, 'at infinity' such a manifold is modelled on a cylinder $\mathbb{R}_+ \times W$, and the metric is assumed to be conical for large $t \in \mathbb{R}_+$, i.e., of the form $dt^2 + t^2 g_W$ for a wedge metric g_W on the cross section W (cf. Definition 2.2 and Section 2.1 below).

The cone $W^\triangle := (\overline{\mathbb{R}}_+ \times W)/(\{0\} \times W)$ itself is interesting as well because of specific corner effects also for $t \to 0$ (near the tip, represented by $\{0\} \times W$, identified with a point v). A calculus for corners of that type is developed in [2]. Operators on a manifold Z with higher edge, modelled on a wedge $W^\triangle \times \Xi$ with edge $\Xi \subseteq \mathbb{R}^p$, have a so called principal edge symbol which consists of a family of operators on $W^\triangle$ parametrised by $(z, \zeta) \in T^*\Xi \setminus 0$, with information both for $t \to 0$ and $t \to \infty$. The main objective of the present paper is the investigation of such edge symbols for $t \to \infty$.

The general background is as follows. Operators on manifolds with 'higher singularities' (e.g., of edge or corner type) may be studied by an iterative approach, parallel to the process of repeatedly forming cones and wedges, combined with global constructions. The cones and wedges are based on already constructed manifolds of lower singularity order. By order zero we understand the smooth case, by order one the case of cones with smooth cross sections or of wedges with such model cones, etc. The program of the (pseudo-differential) analysis is to iterate suitable symbolic hierarchies, associated with the strata of the configuration and to establish corresponding operator algebras. The symbols should be responsible for the ellipticity (or parabolicity) of operators, parametrices, Fredholm property (or invertibility), and regularity and asymptotics of solutions. The problem with higher singularities is to really manage the iteration and to achieve a transparent formalism. Our paper is devoted to one of the typical elements of this approach, namely, the analysis of edge symbols taking values in operators on an infinite non-smooth cone, here of second generation (which means singularities of second order).

In order to illustrate the idea, we first recall some aspects of the simpler case of a smooth manifold with boundary. The operators are identified with boundary value problems. They have a two-component principal symbolic hierarchy $(\sigma_\psi, \sigma_\partial)$, consisting of the interior and the boundary symbol (indicated by subscripts 'ψ' and '∂', respectively). Boundary value problems are connected with many analytical and topological details (e.g., the transmission or violated transmission property, cf. Boutet de Monvel [3], Vishik and Eskin [4], [5], the Atiyah-Bott obstruction for the existence of Shapiro-Lopatinskij boundary conditions, cf. Atiyah and Bott [6], and APS or global projection conditions, cf. Atiyah, Patodi and Singer [7]). Smooth manifolds M with boundary are a subcategory of manifolds W with smooth edges; in the general case the model cones of local wedges may be non-trivial, i.e., of the form $X^\triangle := (\overline{\mathbb{R}}_+ \times X)/(\{0\} \times X)$, for a (smooth compact) base X of non-zero dimension (rather than $\overline{\mathbb{R}}_+$, the inner normal to the boundary of M). As is known from [1] for a manifold W with smooth edge Y, the principal symbolic hierarchy of operators A in the 'edge algebra' consists of two components $(\sigma_\psi, \sigma_\wedge)$, where $\sigma_\wedge$ is the homogeneous principal