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DEGENERATE EVOLUTION INEQUALITIES ON GROUPS OF HEISENBERG TYPE*

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Abstract In this paper the nonexistence for weak solutions of some degenerate evolution inequalities on groups of Heisenberg type is studied.

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1. Introduction

Nonexistence results for weak solutions of degenerate and singular parabolic and hyperbolic inequalities on the Euclidean space R^n have been largely considered by many authors, see Mitidieri and Pohozaev [1-3] and the references therein. Under some suitable assumptions on initial condition, the following problem

$$\begin{cases} u_t - |x|^\sigma \triangle u \geq |u|^q & \text{in} \quad \mathrm{R}^n \times (0, +\infty), \\ u(x,0) = u_0(x) & \text{in} \quad \mathrm{R}^n \end{cases} \tag{1.1}$$

has no nontrivial weak solutions for $0 \leq \sigma < 2$ and $1 < q \leq q_0 = 1 + \frac{2-\sigma}{n-\sigma}$, where q_0 depends on n, σ and the equation. When $\sigma = 2$, the question

$$\begin{cases} u_t - |x|^2 \triangle u \geq |u|^q & \text{in} \quad \mathrm{R}^n\backslash\{0\} \times (0, +\infty), \\ u(x,0) = u_0(x) \neq 0 & \text{in} \quad \mathrm{R}^n\backslash\{0\} \end{cases} \tag{1.2}$$

has no nontrivial weak solutions for $1 < q \leq q_0 = 3$, where q_0 does not depend on the dimension n, provided some hypotheses are satisfied. For this reason $\sigma = 2$ is often taken for the critical case. For hyperbolic problems the same phenomena arises.

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Pohozaev and Vе́ron in [4] investigated the counterpart on the Heisenberg group:

$$\begin{cases} u_t - \triangle_{H^n}(au) \geq \frac{|u|^q}{|\xi|^{\sigma}_{H^n}} & \text{in} \quad H^n \times (0, +\infty), \\ u(\xi, 0) = u_0(\xi) & \text{in} \quad H^n, \end{cases} \tag{1.3}$$

$$\begin{cases} u_{tt} - \triangle_{H^n}(au) \geq \frac{|u|^q}{|\xi|^{\sigma}_{H^n}} & \text{in} \quad H^n \times (0, +\infty), \\ u(\xi, 0) = u_0(\xi) & \text{in} \quad H^n, \\ u_t(\xi, 0) = u_1(\xi) & \text{in} \quad H^n, \end{cases} \tag{1.4}$$

where $a = a(\xi, t)$ is a bounded and measurable function. They proved that for $\sigma < 2$, $1 < q \leq \frac{Q+2-\sigma}{Q}$ and $\int_{H^n} u_0(\xi) d\xi \geq 0$, then no weak solutions of (1.3) exist, and if $\sigma < 2$, $1 < q \leq \frac{Q+1-\sigma}{Q-1}$ and $\int_{H^n} u_1(\xi) d\xi \geq 0$, there exists no weak solution of (1.4).

D'Ambrosio [5] studied the following degenerate inequalities in Heisenberg setting:

$$\begin{cases} u_t - \frac{|\xi|^2_{H^n}}{\psi} \triangle_{H^n}(au) \geq |u|^q & \text{on} \quad H^n \backslash \{0\} \times (0, +\infty), \\ u(\xi, 0) = u_0(\xi) & \text{on} \quad H^n \backslash \{0\}, \end{cases} \tag{1.5}$$

where $a \in$ R, $\psi = \frac{|x|^2 + |y|^2}{|\xi|^2_{H^n}}$, and

$$\begin{cases} u_{tt} - \frac{|\xi|^2_{H^n}}{\psi} \triangle_{H^n}(au) \geq |u|^q & \text{on} \quad H^n \backslash \{0\} \times (0, +\infty), \\ u(\xi, 0) = u_0(\xi) & \text{on} \quad H^n \backslash \{0\}, \\ u_t(\xi, 0) = u_1(\xi) & \text{on} \quad H^n \backslash \{0\}, \end{cases} \tag{1.6}$$

where $a \in L^{\infty}(H^n \times [0, +\infty))$, $\psi = \frac{|x|^2 + |y|^2}{|\xi|^2_{H^n}}$. The author found that in Heisenberg setting these are critical cases too. As well as in Euclidean case, the exponent q_0 is $q_0 = 3$ for (1.5) and (1.6).

The aim of this paper is to research into nonexistence theorems for weak solutions of some degenerate evolution inequalities on groups of Heisenberg type:

$$\begin{cases} u_t - \frac{d^2}{\psi} \triangle_G(au) \geq |u|^q & \text{on} \quad G \backslash \{(0, 0)\} \times (0, +\infty), \\ u(x, y, 0) = u_0(x, y) & \text{on} \quad G \backslash \{(0, 0)\}, \end{cases} \tag{1.7}$$

where $a \in$ R, and

$$\begin{cases} u_{tt} - \frac{d^2}{\psi} \triangle_G(au) \geq |u|^q & \text{on} \quad G \backslash \{(0, 0)\} \times (0, +\infty), \\ u(x, y, 0) = u_0(x, y) & \text{on} \quad G \backslash \{(0, 0)\}, \\ u_t(x, y, 0) = u_1(x, y) & \text{on} \quad G \backslash \{(0, 0)\}, \end{cases} \tag{1.8}$$

where $a \in L^{\infty}(G \times [0, +\infty))$, Δ_G, d and ψ are defined in the next section.

Heisenberg type groups are an interesting class of Carnot groups of step two in connection with hypoellipticity questions. Such groups, which were introduced by Kaplan [6] in 1980, constitute a direct generalization of Heisenberg groups and are