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AN INITIAL-BOUNDARY-VALUE PROBLEM FOR THE MODIFIED KORTEWEG-DE VRIES EQUATION IN A QUARTER PLANE

Shen Caixia
(Graduate School of China Academy of Engineering Physics, Beijing, 100088, China)
(E-mail: shencaixia@tom.com)
Guo Boling
(Institute of Applied Physics and Computational Mathematics, Beijing, 100088, China)
(E-mail: gbl@mail.iapcm.ac.cn)
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Abstract In this paper, we obtain some linear estimates, trilinear estimates. Through these estimates , we prove the local wellposeness of modified Korteweg-de Vries equation in a quarter plane.

Key Words the modified Korteweg-de Vries equation; initial-boundary-value problem; well-posedness.

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1. Introduction

The KdV equations are of the form:

$$u_t + u_{xxx} + P(u)_x = 0$$

where $u(x,t)$ is a function of one space and one time variable, and $P(u)$ is some polynomials of u. Historically, these types of equations first arose in the study of 2D shallow wave propagation, but have since appeared as limiting cases of many dispersive models.

When $P(u) = Cu^{k+1}$, the equation is referred to as generalized KdV of order k, or gKdV-k, gKdV-1 is the original Korteveg-de Vries equation, gKdV-2 is the modified KdV equation (mKdV).

There are a lot of works on the KdV equation for the following pure initial problem (0.1) [1-7]

$$\begin{cases} u_t + u_x + u_{xxx} + uu_x = 0, \ x, \ t \geq 0 \\ u(x,0) = \phi(x) \end{cases} . \tag{0.1}$$

The problem (0.1) is locally well-posed for initial value ϕ in the space H^s for $s > -\frac{3}{4}$ in Kenig ,Ponce ,and Vega [2] ,and Tao proved the problem (0.1) is global well-possed for initial value ϕ in the space H^s for $s > -\frac{3}{4}$ in [7] .These are the best results up to now.

$$\begin{cases} u_t + u_x + u_{xxx} + uu_x = 0,\ x,\ t \geq 0 \\ u(x,0) = \phi(x) \\ u(0,t) = h(t) \end{cases}. \tag{0.2}$$

As for the KdV equation for initial-boundary problem (0.2) ,we have the following : It is locally well-posed for initial data in the space $H^s(R^+)$and boundary data h in the space $H_{loc}^{(s+1)/3}$ satisfying certain compatibility conditions for $s > 3/4$, whereas global well-posed holds for $\phi \in H^s(R^+), h \in H_{loc}^{(7+3s)/12}$ when $1 \leq s \leq 3$and for $\phi \in H^s(R^+), h \in H_{loc}^{(s+1)/3}$ when $s \geq 3$.

In this paper we want to discuss the following problem

$$\begin{cases} u_t + u_x + u_{xxx} + u^2u_x = 0,\ x,\ t \geq 0 \\ u(x,0) = \phi(x) \\ u(0,t) = h(t) \end{cases}, \tag{1.1}$$

i.e. we shall discuss non-homogeneous-value problem for the mKdV equation . We may use some results in [8]directly , and use the modern theory in [9]for the pure initial-value problem posed on R .

The main theory in this paper are the following:

Theorem 1 *Let $T > 0$.For a pair of functions $\phi \in H^{1/4}(R^+), h \in H^{(1+1/4)/3}(0,T)$, there exists a $T^* \in (0,T]$ depending on $\|\phi\|_{H^{1/4}(R^+)} + \|h\|_{H^{(1+1/4)/3}(0,T)}$such that* (4.1) *admits a unique solution u and $\Omega^{T^*}(u) \leq \infty$.*

Theorem 2 *Let $T > 0, s \geq \frac{1}{4}$. For a pair of functions $\phi \in H^s(R^+), h \in H^{(1+s)/3}(0,T)$, there exists a $T^* \in (0,T]$ depending on $\|\phi\|_{H^s(R^+)} + \|h\|_{H^{(1+s)/3}(0,T)}$such that* (4.1)*admits a unique solution u and $\Omega^{T^*}(u) \leq \infty$, where $\Omega^T(\cdot)$ will be explained in Section 3.*

The proof of Theorem 2 is omitted.

2. The Linear Estimates

We consider the non-homogeneous boundary value problem

$$\begin{cases} u_t + u_x + u_{xxx} = 0, \quad x,t \geq 0 \\ u(x,0) = 0 \\ u(0,t) = h(t) \end{cases}. \tag{2.1}$$

By use of the Laplace transform and the theory of ordinary difference equations, we get the solution of (2.1) is:

$$u(x,t) = [W_b(t)h](x) = [U_b(t)h](x) + \overline{[U_b(t)h](x)}, x, t \geq 0,$$