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COEXISTENCE STATES OF A STRONGLY COUPLED PREY-PREDATOR MODEL*

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Abstract This paper is concerned with a strongly coupled prey-predator model with homogeneous Dirichlet boundary conditions. The existence and uniqueness of coexistence states are discussed.

Key Words Strongly coupled prey-predator model; coexistence states; bifurcation; uniqueness.

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1. Introduction

Let Ω be a bounded domain in $\mathbf{R}^N (N \geq 1)$ with smooth boundary $\partial\Omega$. We consider the following strongly coupled prey-predator model:

$$\begin{cases} -\Delta\Big[(d_1+\alpha v)u\Big] = u(a_1 - b_1 u - c_1 v) & \text{in } \ \Omega, \\ -\Delta\Big[\Big(d_2 + \dfrac{\gamma}{\beta+u}\Big)v\Big] = v(a_2 + b_2 u - c_2 v) & \text{in } \ \Omega, \\ u = v = 0 & \text{on } \ \partial\Omega, \end{cases} \tag{1.1}$$

where α and γ are nonnegative constants; $d_i, a_i, b_i, c_i (i = 1, 2)$ and β are also constants, they are all positive except for a_2 which may be non-positive. In this model, $u(x)$ and $v(x)$ represent the population densities of prey and predator species respectively, which are interacting and migrating in the same habitat Ω. The boundary condition means that the habitat Ω is surrounded by a hostile environment. The diffusion terms can be written as

$$\text{div}\{(d_1+\alpha v)\nabla u + \alpha u \nabla v\} \text{ and } \text{div}\{-\frac{\gamma v}{(\beta+u)^2}\nabla u + (d_2 + \frac{\gamma}{\beta+u})\nabla v\}.$$

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Thus, $J_{u,x} := -\{(d_1+\alpha v)\nabla u + \alpha u \nabla v\}$ and $J_{v,x} := -\{-\gamma v/(\beta+u)^2\nabla u + (d_2+\gamma/(\beta+u))\nabla v\}$ are the diffusive fluxes of u and v in x-direction respectively. The terms $d_1+\alpha v$ and $d_2+\gamma/(\beta+u)$ represent the "self-diffusion". The terms αu and $\gamma v/(\beta+u)^2$ are the "cross-diffusion". The term $\alpha u \geq 0$ implies that the flux of u in x-direction is directed toward decreasing population of v, i.e. the prey avoids the predator, while $\gamma v/(\beta+u)^2 \leq 0$ implies that the flux of v in the x-direction is directed toward increasing population of u, i.e. the predator chases the prey. The function $\gamma v/(\beta+u)^2$ is decreasing with respect to u, meaning that, due to more and prey, the chaseable capacity of predator is decreasing with the enhanced resistance of prey. The above model means that, in addition to the the dispersive force, the diffusion also depends on population pressure from other species. See [1] for a more detailed discussion on biological models, and also [2] for a survey on the mathematical developments.

By the scaling

$$(\tilde{u},\tilde{v},\tilde{\alpha},\tilde{\beta},\tilde{\gamma},a,(1+\frac{\tilde{\gamma}}{\tilde{\beta}})b,c,m) = (\frac{b_1}{d_1}u, \frac{c_2}{d_2}v, \frac{d_2}{c_2 d_1}\alpha, \frac{b_1}{d_1}\beta, \frac{b_1}{d_1 d_2}\gamma, \frac{a_1}{d_1}, \frac{a_2}{d_2}, \frac{c_1 d_2}{c_2 d_1}, \frac{b_2 d_1}{b_1 d_2}),$$

and omit the '$\tilde{}$' sign, (1.1) is reduced to the following problem:

$$\begin{cases} -\Delta[(1+\alpha v)u] = u(a-u-cv) & \text{in } \Omega, \\ -\Delta\Big[\Big(1+\dfrac{\gamma}{\beta+u}\Big)v\Big] = v\big((1+\dfrac{\gamma}{\beta})b+mu-v\big) & \text{in } \Omega, \\ u=v=0 & \text{on } \partial\Omega. \end{cases} \tag{P}$$

Since we are concerned with the solutions of (P) as representing steady states of a prey-predator population dynamic models, only nonnegative solution couples (u,v) of (P) are of practical interest. Besides the trivial solution $(0,0)$, the semitrival solutions which have one of their components identically zero, there are coexistence states, namely, positive solutions, having both components strictly positive. In this paper, we address the problem of the existence and uniqueness of coexistence states for (P).

When there are no strongly coupled effects ($\alpha=\gamma=0$), (P) is reduced to the classical Lotka-Volterra prey-predator diffusion model, which has been extensively discussed by many authors (e.g., [3–9]).Especially, they have a necessary and sufficient condition for the existence of a positive solution([5, 6]). So it is possible to determine completely the coexistence region in a parameter space (a,b)([6]). For a one-dimensional domain, the existence and uniqueness of coexistence states are given([7]). With strongly coupled effects, in the same sense of biology, the system under homogeneous Neumann boundary conditions, there were some works which discussed the existence and the nonexistence of nonconstant positive solutions by the degree theory and bifurcation theory (e.g., [10, 11]). The problem with the aggregation term $\Delta[(1+\beta u)v]$ (instead of $\Delta\big[(1+\frac{\gamma}{\beta+u})v\big]$ in (P)) has been studied in [12, 13].

In this model, we introduce two unknown functions U and V as follows

$$U=(1+\alpha v)u \quad \text{and} \quad V=(1+\frac{\gamma}{\beta+u})v. \tag{1.2}$$