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NEUMANN BOUNDARY VALUE PROBLEM FOR THE LANDAU-LIFSHITZ EQUATION

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Abstract In this paper, we prove that there exists a unique global smooth solution for the homogeneous Neumann boundary value problem of the Landau-Lifschitz equation if the initial function is smooth.

Key Words Landau-Lifschitz Equation; difference method; existence and uniqueness.

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1. Introduction

In this paper we study the homogeneous Neumann boundary value problem for the Landau-Lifshitz equation of the form

$$u_t = u \times u_{xx}, \qquad (x,t) \in (0,l) \times \mathbb{R}^+ \tag{1.1}$$

$$u|_{t=0} = \Phi(x), \qquad x \in (0,l) \tag{1.2}$$

$$\frac{\partial u}{\partial x}|_{x=0} = 0, \quad \frac{\partial u}{\partial x}|_{x=l} = 0, \quad t \geqslant 0 \tag{1.3}$$

where $u = u(x,t); (0,l) \times \mathbb{R}^+ \longrightarrow S^2$ is the unknown function,$\times$ denotes the cross product in $\mathbb{R}^3$.

The Landau-Lifshitz equation which describes the evolution of spin fields in continuum ferromagnets bears a foundamental role in the understanding of nonequilibrium magnetism [1]. The equation (1.1) is the result of the Landau-Lifshitz equation after neglecting lower order terms and Gibert damping term. For the one-dimensional

Cauchy problem and nonhomogeneous boundary value problem of (1.1), the existence and uniqueness of global regular solution have been proved in [2,3] by applying the vanishing viscosity method. Except the results of [4] for 2-dimensional radial symmetric Landau-Lifshitz equation with Neumann boundary value in exterior domain, many basic mathematical questions of multidimensional equation (1.1) are still an open problem. For the problem (1.1)-(1.3), we prove the following result by use of different tecniques from [2,3].

Theorem 1 *For initial data $\Phi(x)$ with $\Phi(x) \in S^2$ and $\Phi_x \in H^m[0,l]$ ($\forall m \geq 1$),there exists a solution of the problem (1.1)-(1.3) such that $u \in S^2$ and for all time $T > 0$*

$$u_{x^{k_1}t^{k_2}} \in C(0,T;L^2[0,l]),$$

where $1 \leq 2k_2 + k_1 \leq m+1$.Moreover, for all integer $m \geqq 3$,the solution is unique.

This paper is organized as follows. In Section 2, we prove the existence and uniqueness of the local solution for the problem (1.1)-(1.3).In Section 3,a prior estimates are established for the solution of the problem (1.1)-(1.3).Based on a prior estimates obtained ,in Section 4,we prove theorem 1.1.

Different positive constants,unless especially noted,will be denoted by the same letter C.

2. Local Existence

Our aim in this subsection is to construct the local solution (in time t) of the problem (1.1) as limits,when h tends to zero,of the sequence $\{u_h\}$ satisfying the following ordinary differential-difference system

$$\frac{du_j}{dt} = u_j \times \frac{\triangle_+\triangle_- u_j}{h^2}, j = 1,2,\cdots,J \tag{2.1}$$

with initial data

$$u_j(0) = \phi_j = \phi(jh), j = 1,\cdots,J-1, \tag{2.2}$$

and boundary conditions

$$\frac{\triangle_+ u_0}{h} = 0, \frac{\triangle_- u_J}{h} = 0, \tag{2.3}$$

where h is step size,$0 < h < 1, h = \frac{l}{J}, u_j = u(jh,t)$ $(j = 0,1,\cdots J)$, $\triangle_+$ and $\triangle_-$ denote the forward and backward difference operators respectively. It is well-known that there exists a local smooth solution of the problem $(2.1)-(2.3)$ $u_h = \{u_j = u(jh,t) \mid j = 0,1,\cdots,J\}$.In order to verify the local existence of the problem $(1.1)-(1.3)$,it suffices to get the uniform a prior estimates of u_h with respect to h .