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STABILITY OF P_2 METHODS FOR NEUTRON TRANSPORT EQUATIONS*

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Abstract In this paper the P_2 approximation to the one–group planar neutron transport theory is discussed. The stability of the solutions for P_2 equations with general boundary conditions, including the Marshak boundary condition, is proved. Moreover, the stability of the up–wind difference scheme for the P_2 equation is demonstrated.
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1. Introduction

We shall consider the following one–group planar transport problem

$$\frac{\partial\psi}{\partial t}+\mu\frac{\partial\psi}{\partial x}+\sigma(x,t)\psi(x,\mu,t)$$

$$=\int_{-1}^{1}\sigma_s(x,t,\mu,\mu')\psi(x,\mu',t)d\mu'+\frac{1}{2}Q(x,t),\ \ 0<x<l,-1\le\mu\le 1,0<t\le T, \quad (1)$$

$$\psi(x,0,\mu)=0,\ \ 0\le x\le l,-1\le\mu\le 1, \quad (2)$$

$$\psi(0,t,\mu)=\psi(0,t,-\mu),\ \ 0\le t\le T,0<\mu\le 1, \quad (3)$$

$$\psi(l,t,\mu)=0,\ \ 0\le t\le T,-1\le\mu<0, \quad (4)$$

where $\psi(x,t,\mu)$ is the angular flux at position x at time t traveling with constant speed $v=1$ in direction $\mu=\cos\theta$, $\sigma(x,t)$ is the total cross section, $Q(x,t)$ is the interior source term, and the scattering cross section is given by

$$\sigma_s(x,t,\mu,\mu')=\sum_{n=0}^{\infty}\frac{2n+1}{2}P_n(\mu)P_n(\mu')\sigma_{sn}(x,t), \quad (5)$$

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where $P_n(\mu)$ is the Legendre polynomial of order n, and σ_s satisfies

$$\int_{-1}^{1} \sigma_s(x,t,\mu,\mu')d\mu' = 1. \tag{6}$$

By introducing the P_2 approximation as follows

$$\psi(x,t,\mu) \approx \sum_{n=0}^{2} \frac{2n+1}{2} P_n(\mu)\psi_n(x,t), \tag{7}$$

we obtain the P_2 equation

$$\frac{\partial \vec{\psi}}{\partial t} + A_0 \frac{\partial \vec{\psi}}{\partial t} + B_0\vec{\psi} = \vec{q}_0, \tag{8}$$

where $\vec{\psi} = (\psi_0, \psi_1, \psi_2)'$, $\vec{q}_0 = (Q, 0, 0)'$, the prime represents the transposition, and

$$A_0 = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & \frac{2}{5} & 0 \end{pmatrix}, \quad B_0 = \ \mathrm{diag}\ (\sigma_{a0}, \sigma_{a1}, \sigma_{a2}),$$

$\sigma_{ai} = \sigma_{ai}(x,t) = \sigma(x,t) - \sigma_{si}(x,t)$ for $i = 0, 1, 2$.

The initial condition for P_2 equation is

$$\psi_i(x,0) = 0, \quad 0 \le x \le l, \quad (i = 0, 1, 2). \tag{9}$$

The boundary condition at $x = 0$ is

$$\psi_1(0,t) = 0, \quad 0 \le t \le T, \tag{10}$$

which is corresponding to the reflective boundary condition (3).

Historically, the P_2 approximation has been viewed as a questionable approximation to the transport equation since there is an ambiguity in the prescription of boundary conditions corresponding to the vacuum boundary condition (4) and incoming boundary condition. Many papers have been devoted to the derivation of the boundary condition for the stationary and nonstationary P_2 theory (e.g.,see [1–4]) based on variational analysis or asymptotic limit analysis of boundary layer. The boundary conditions so obtained are of the following form

$$d_0\psi_0(x,t) + d_1\psi_1(x,t) + d_2\psi_2(x,t) = 0, \quad x = l, \tag{11}$$

where d_i $(i = 0, 1, 2)$ are non–zero constants. Different papers give different values of d_i $(i = 0, 1, 2)$. We note that, by taking $d_0 = \frac{1}{2}$, $d_1 = -1$ and $d_2 = \frac{5}{8}$, (11) reduces to the well–known Marshak boundary condition.