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A CLASS OF SINGULARLY PERTURBED SEMILINEAR ELLIPTIC EQUATIONS*

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Abstract The singularly perturbed problem for the semilinear elliptic equations is considered. Under appropriate conditions, by using the comparison theorem, the existence and asymptotic behavior of solution for the boundary value problems are studied.

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Consider the singularly perturbed problem in a strip domain $\Omega_n \equiv \{x \mid 0 < x_n < a\}$as follows:

$$\varepsilon Lu + L_1 u = f(x, u, Tu, \varepsilon), \tag{1}$$

$$u = g_1(x_1, \cdots, x_{n-1}), x_n = 0, \tag{2}$$

$$u = g_2(x_1, \cdots, x_{n-1}), x_n = a, \tag{3}$$

where

$$L \equiv \sum_{j,k=1}^{n} a_{jk}(x) \frac{\partial^2}{\partial x_j \partial x_k} + \sum_{j=1}^{n} b_j(x) \frac{\partial}{\partial x_j},$$

$$\sum_{j,k=1}^{n} a_{jk}(x) \xi_j \xi_k \geq \lambda \sum_{j=1}^{n} \xi_j^2, \ \ \forall \xi_j \in R, \lambda > 0,$$

$$L_1 = -\sum_{j=1}^{n} c_j(x) \frac{\partial}{\partial x_j},$$

$$Tu = \int_{\Omega} K(x, y) u(y) dy,$$

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where ε is a positive parameter,$x = (x_1, x_2, \cdots, x_n) \in \overline{\Omega}_n$. The authors have studied a class of singularly perturbed boundary value problems for the elliptic equations in [1-4]. This paper involves singularly perturbed problem in an unbounded domain.

Assume that

$[H_1]$ the coefficients of L and L_1 are bounded smooth functions in $\overline{\Omega} \equiv \{0 \leq x_n \leq a\}$;

$[H_2]$ f, g_1, g_2 and their derivatives until m-th order are bounded continuous functions with regard to their variables;

$[H_3]$ $c_n(x) > 0$, $\min\{f_y(x, y, z, \varepsilon), f_z(x, y, z, \varepsilon)\} \geq b_0 > 0, \int_\Omega K(x, y)dy \geq M, M$ is a positive constant;

$[H_4]$ the reduced problem of (1)-(3)

$$L_1 u = f(x, u, Tu, 0),$$

$$u = g_1(x_1, \cdots, x_{n-1}), x_n = 0$$

has a bounded smooth solution U_0 in $\overline{\Omega}_n$.

We now construct the formal asymptotic solution of the problem (1)-(3) being

$$U \sim \sum_{i=0}^{\infty} U_i \varepsilon^i. \tag{4}$$

Substituting (4) into (1),developing f in ε,equating coefficients of like powers of ε respectively, we obtain

$$L_1 U_i - f_y(x, U_0, TU_0, 0)U_i - f_z(x, U_0, TU_0, 0)TU_i = -LU_{i-1} + F_i,$$

$$U_i = 0, x_n = 0, i = 1, 2, \cdots,$$

where F_i are determined functions of $U_\gamma (\gamma \leq i-1)$, and their constructions are omitted. The above and below, the values of terms for the negative subscript are zero. From above linear equation and $c_n(x) > 0$, we can solve U_i successively. From (4),we obtain the outer solution U for the original problem. But it may not satisfy the boundary condition (3), so we need to construct the boundary layer term V near $x_n = a$.

We lead into variables of multiple scales [5]:

$$\tau = \frac{h(x_1, \cdots, x_n)}{\varepsilon}, \rho = x_n, \tag{5}$$

where $h(x_1, \cdots, x_n)$ is a function to be determined. For convenience, we still substitute x_n for ρ below. From (5),we have

$$L = \frac{1}{\varepsilon^2} K_0 + \frac{1}{\varepsilon} K_1 + K_2,\ L_1 = \frac{1}{\varepsilon} P_0 + P_1, \tag{6}$$

where

$$K_0 = a_{nn} h_{x_n}^2 \frac{\partial^2}{\partial \tau^2},$$