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A Class of Periodic Solutions of One-Dimensional Landau-Lifshitz Equations

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Abstract. We study one-dimensional Landau-Lifshitz Equations and give the sufficient and necessary conditions for the existence of a class of periodic solutions.

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1 Introduction

Here we consider the following Schrödinger type flow

$$\frac{\partial u}{\partial t}=u\times(\Delta u+\nabla h\nabla u+\lambda u_3e_3), \tag{1.1}$$

where $u(t,x)=(u_1,u_2,u_3):\mathbb{R}\times\mathbb{R}^n\to\mathbb{S}^2\subseteq\mathbb{R}^3$, $h\in C^\infty(\mathbb{R}^n)$, $\lambda\in\mathbb{R}$ and $e_3=(0,0,1)$ denotes the north pole. In fact, above equation is just Hamilton system with respect to the following functional

$$E_h(u)=\frac{1}{2}\int_{\mathbb{R}^n}|\nabla u|^2e^hdx+\frac{\lambda}{2}\int_{\mathbb{R}^n}(1-u_3^2)e^hdx. \tag{1.2}$$

Let us recall some results on the Schrödinger type flow (1.1). Despite a great deal of mathematical efforts, some basic mathematical issues such as the global well-posedness

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and global-in-time asymptotics for (1.1) are still unclear. Therefore, some authors (see [5, 9, 10]) focused on finding some soliton solutions to (1.1). As h is a constant function, equation (1.1) is just Landau-Lifshitz equation with easy-axis anisotropic, for simplicity, denoted by **LLEE**. In [5], Gustafson and Shatah studied the time-periodic solitary wave solutions (also called vortex solutions) to **LLEE** in two spatial dimensions with $\lambda > 0$ (see also [6]), while Lin and Wei ([9]) constructed traveling wave solutions with $\lambda < 0$. However, Kollar ([8]) showed the nonexistence of vortex solutions to the same problem with $\lambda=0$ (See also [10]).Then limited work has been done in seeking for soliton solutions to (1.1) for $n=1$ or $n\geq 3$.

The main purpose of this paper is to obtain some soliton solutions to equation (1.1) on one-dimensional space, i.e. $n=1$. Specifically, we look for a solution of the following form

$$u(t,x)=(\sin\alpha(x)\cos(\omega t),\sin\alpha(x)\sin(\omega t),\cos\alpha(x)), \tag{1.3}$$

where $\alpha\in C^\infty(\mathbb{R})$ and $\omega\in\mathbb{R}$ is the angular velocity. After a simple calculation, the equation (1.1) reduces to an ordinary differential equation (ODE) of $\alpha(x)$

$$\alpha''(x)+h'(x)\alpha'(x)=g(\alpha(x)), \tag{1.4}$$

where

$$g(\cdot)=\lambda\sin(\cdot)\cos(\cdot)+\omega\sin(\cdot). \tag{1.5}$$

In view of physical background, we are more concerned with such solutions with finite energy, i.e. $|E_h(u)|<\infty$, which is equivalent to the following boundary condition of $\alpha(x)$

$$\lim_{x\to\infty}\alpha(x)=k\pi \quad \text{and} \quad \lim_{x\to-\infty}\alpha(x)=(k+2l)\pi,$$

where $k,l\in\mathbb{Z}$.

For simplicity, we only consider the case $k=1$ and $l=-1$, i.e.

$$\lim_{x\to\infty}\alpha(x)=\pi \quad \text{and} \quad \lim_{x\to-\infty}\alpha(x)=-\pi. \tag{1.6}$$

In order to find solutions to problem (1.4)-(1.6), Let us consider the following boundary value problem

$$\text{(BVP)}\begin{cases}\alpha''(x)+h'(x)\alpha'(x)=g(\alpha(x)), & 0\leq\alpha(x)\leq\pi, \quad x\in(0,\infty),\\ \alpha(0)=0, \quad \alpha(\infty)=\pi,\end{cases}$$

where $h\in C_0^\infty(\mathbb{R})$ is an even function. It's easy to verify that if $\alpha(x)$ is a solution to (BVP), then

$$\overline{\alpha}(x)=\begin{cases}\alpha(x), & x\in[0,\infty),\\ -\alpha(-x), & x\in(-\infty,0)\end{cases} \tag{1.7}$$