

The Traveling Wave of Auto-Catalytic Systems-Monotone and Multi-Peak Solutions

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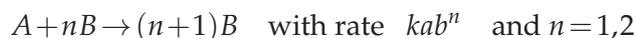
Abstract. This article studies propagating wave fronts of a reaction-diffusion system modeling an isothermal chemical reaction $A+2B\rightarrow 3B$ involving two chemical species, a reactant A and an auto-catalyst B , whose diffusion coefficients, D_A and D_B , are unequal due to different molecular weights and/or sizes. Explicit bounds c_* and c^* that depend on D_B/D_A are derived such that there is a unique travelling wave of every speed $c\geq c^*$ and there does not exist any travelling wave of speed $c < c_*$. Furthermore, the reaction-diffusion system of the Gray-Scott model of $A+2B\rightarrow 3B$, and a linear decay $B\rightarrow C$, where C is an inert product is also studied. The existence of multiple traveling waves which have distinctive number of local maxima or peaks is shown. It shows a new and very distinctive feature of Gray-Scott type of models in generating rich and structurally different traveling pulses.

AMS subject classifications: 34C20, 34C25, 92E20

Key words: Cubic autocatalysis, travelling wave, minimum speed, Gray-Scott, multi-peak waves.

1 Introduction

Autocatalytic chemical reaction of the form



between two chemical species A and B , appears in many chemical wave models of excitable media from the idealized Brusselator to real-world clock reactions such as Belousov-Zhabotinsky reaction, the Briggs-Rauscher reaction, the Bray-Liebhafsky reaction and the iodine clock reaction. In that setting, their importance was recognized pretty early [13, 14, 29].

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More recently, in various models of biological pattern formation of Turing type, for the purpose of replicating experimental results in early 1990s, whether it is CIMA or Gray-Scott [20,22] chemical reaction of the form



with C an inert chemical species, plays a significant role. In particular, in Gray-Scott model with feeding, self-replicating traveling pulse (traveling wave) is the most exciting and not completely understood phenomenon [9–11, 18].

In this work, we study the traveling wave problem of autocatalytic chemical reaction $A + nB \rightarrow (n+1)B$, which, after simple non-dimensionalization results in the reaction-diffusion system,

$$(I) \begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - uv^n, \\ \frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + uv^n, \end{cases}$$

as well as that of chemical reaction $A + nB \rightarrow (n+1)B$, and $B \rightarrow C$, which has the governing equations

$$(II) \begin{cases} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - uv^n, \\ \frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + uv^n - kv^m. \end{cases}$$

Here, D a positive constant is the ratio of diffusion coefficients of chemical species B to that of A , $n \geq 1$ is a positive constant not necessarily an integer, and kv^m describes the rate of $B \rightarrow C$, with k and $m \geq 1$ both positive constants. We assume throughout that $1 \leq m \leq n$.

For a traveling wave solution to (I), $u(x,t) = u(z)$, $v(x,t) = v(z)$, where $z = x - ct$, the governing ODE system is:

$$\begin{cases} u'' + cu' - uv^n = 0, \\ Dv'' + cv' + uv^n = 0, \end{cases} \quad (1.1)$$

where $c > 0$ is a constant. Assuming

$$\lim_{z \rightarrow -\infty} (u, v) = (0, a), \quad a > 0,$$

the addition of the two equations and integration on $(-\infty, z]$ yield

$$u' + Dv' + c(u + v - a) = 0. \quad (1.2)$$