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Chebyshev Spectral Method for Volterra Integral Equation with Multiple Delays

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Abstract. Numerical analysis is carried out for the Volterra integral equation with multiple delays in this article. Firstly, we make two variable transformations. Then we use the Gauss quadrature formula to get the approximate solutions. And then with the Chebyshev spectral method, the Gronwall inequality and some relevant lemmas, a rigorous analysis is provided. The conclusion is that the numerical error decay exponentially in L^∞ space and $L^2_{\omega^c}$ space. Finally, numerical examples are given to show the feasibility and effectiveness of the Chebyshev spectral method.

AMS subject classifications: 65R20, 45E05

Key words: Volterra integral equation, multiple delays, Chebyshev spectral method, Gronwall inequality, convergence analysis.

1 Introduction

In this paper, we consider the Volterra integral equation with multiple delays of the form

$$y(t)-\sum_{l=1}^{M}\int_0^t k_l(t,\xi)y(a_l\xi)d\xi=f(t), \tag{1.1}$$

where the unknown function $y(t)$ is defined on $0\leqslant t\leqslant T<+\infty$. The source function $f(t)$ and kernel function $k_l(t,\xi)(l=1,2,...,M)$ are given sufficiently smooth functions, with the condition that M is a given natural number, $0<a_l\leqslant 1$.

These kinds of equations arise in many areas, such as the Mechanical problems of physics, the movement of celestial bodies problems of astronomy and the problem of biological population original state changes. It is also applied to network reservoir, storage

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system, material accumulation, etc., and solve a lot problems from mathematical models of population statistics, viscoelastic materials and insurance abstracted. Due to the significance of these equations which have played in many disciplines, they must be solved efficiently with proper numerical approach. In recent years, these equations have been extensively researched, such as collocation methods [3–5, 21], Taylor series methods [10], linear multistep methods [14], spectral analysis [1,2,7–9,11,18–20]. In fact, spectral methods have excellent error properties called "exponential convergence" which is the fastest possible. There are many also many spectral methods to solve Volterra integral equations, for example Legendre spectral-collocation method [18], Jacobi spectral-collocation method [8], spectral Galerkin method [20], Chebyshev spectral method [11] and so on. As the Chebyshev points are easier to be obtained than in [2], in this paper we are going to use the Chebyshev spectral method to deal with the Volterra integral equation with multiple delays. Meanwhile the error estimate of the $L^2_{\omega^c}$ norm is observed in our article, while in I. Ali, H. Brunner and T. Tang′s is not. The third difference is that I. Ali, H. Brunner and T. Tang′s article only has two delay terms, while our article has M delay terms. Compared to the work by Zhang Ran in [22], the novelty is that the delay term in their article is in the integral term while in ours the delay terms are in the integrand functions. In a word, we provide rigorous error analysis by Chebyshev spectral method for the Volterra integral equation with multiple delays that theoretically justifies the spectral rate of convergence in this paper. Numerical tests are also presented to verify the theoretical result.

We organize this paper as follows. In Section 2, we introduce the Chebyshev spectral method. Some knowledge which is important for the derivation of the mail result is given in the next section. We carry out the convergence analysis in Section 4 and Section 5 contains numerical tests which are illustrated to confirm the theoretical result. In the end a conclusion is given in Section 6.

Throughout the paper C denotes a positive constant that is independent of N, but depends on other given conditions.

2 Chebyshev spectral method

In this section, we review Chebyshev spectral method. Firstly we use the variable transformations as follow

$$t=\frac{T(1+x)}{2}, \qquad \xi=\frac{T(1+s)}{2},$$

and if we note that

$$u(x)=y\Big(\frac{T(1+x)}{2}\Big), \quad \hat{k}_l(x,s)=\frac{T}{2}k_l\Big(\frac{T(1+x)}{2},\frac{T(1+s)}{2}\Big), \quad g(x)=f\Big(\frac{T(1+x)}{2}\Big),$$

then (1.1) can be written as

$$u(x)-\sum_{l=1}^{M}\int_{-1}^{x}\hat{k}_l(x,s)u(a_ls+a_l-1)ds=g(x), \quad x\in[-1,1]. \tag{2.1}$$