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Time-Varying Moving Average Model for Autocovariance Nonstationary Time Series

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Abstract

In time series analysis, fitting the Moving Average (MA) model is more complicated than Autoregressive (AR) models because the error terms are not observable. This means that iterative nonlinear fitting procedures need to be used in place of linear least squares. In this paper, Time-Varying Moving Average (TVMA) models are proposed for an autocovariance nonstationary time series. Through statistical analysis, the parameter estimates of the MA models demonstrate high statistical efficiency. The Akaike Information Criterion (AIC) analyses and the simulations by the TVMA models were carried out. The suggestion about the TVMA model selection is given at the end. This research is useful for analyzing an autocovariance nonstationary time series in theoretical and practical fields.

Keywords: MA Model; Autocovariance; Parameter Estimation; Simulation; Model Selection

1 Introduction

For the past decades, time series analysis has become a highly developed subject, and there are now well-established methods for fitting a wide range of models to time series data as in the books [1-4] and in the articles [5-8]. However, none of these studies focused on autocovariance nonstationary time series. Virtually all the established methods rest on one fundamental assumption, namely, that the process is *autocovariance* stationary, or locally stationary. At least the statistical characteristics of the nonstationary processes class are changing *smoothly* over time. The nonstationarity of the time series means at least one statistical characteristic is variant with time points. The mean (or trend) nonstationary time series can usually be reduced to mean stationary by some simple transformation, such as the difference method. The autocovariance is one of the most important statistical characteristics of the nonstationary time series [9] and can be used as an important index when evaluating simulation by some time series analysis models. Needless to say, for a single time series sampled from some time series, the assumption of autocovariance stationarity is a mathematical idealization which, in some cases, may be valid only as an approximation to the real situation. In practical applications, the most one could hope for is, over the

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observed time interval, the series would not depart *too far* from autocovariance stationarity for the results of the analysis to be valid.

It is not difficult to see why the notion of autocovariance stationarity is such an appealing one. It endows the process with *statistical stability* so that autocovariances originally defined as ensemble averages can be estimated from a single realization by computing the corresponding time domain averages. (Strictly speaking, the property to which we are referring here is *ergodicity* rather than stationarity, but in practical terms the two ideas are very closely related.) However, like virtually all mathematical concepts, stationarity is an idealization, and in practice, it can at best be expected to hold only as an approximation. In order to carry out meaningful statistical analysis on a single time series, the time series is usually considered to be in some sense locally stationary although globally nonstationary [3, 5, 8]. It is clearly of interest, therefore, to examine if any type of the analysis is available for those cases where the assumption of autocovariance nonstationarity becomes realistic.

Usually, it is hard to statistically analyze a nonstationary time series for its complexity. Nonstationary models offer greater complexity than stationary models, and the statistical problems of model identification and parameter estimation are similarly more intricate. Experience gained so far has shown that even simple nonstationary models can capture examples of time series behavior which would be impossible to describe with stationary models. For a zero mean autocovariance nonstationary time series with finite length, the full order Time-varying Parameter Autoregressive (TVPAR) model with time varying AR coefficients and variances of residuals [10] was used for simulation [11] and pattern recognition [12]. The term TVPAR model was adopted to differentiate from the TVAR model in which the variance of residual is time invariant [13], at least in a time segment. There are three TVPAR models: full order TVPAR (TVPAR(T)), time-unvarying order TVPAR (TVPAR(p)), and time-varying order TVPAR (TVPAR(p_t)) [14]. In this paper, three time-varying moving average (TVMA) models are presented and analyzed by means of AIC [15], and TVMA model selection is suggested. The AIC is a measure of the relative quality of a statistical model, for a given set of data. As such, AIC provides a means for model selection. In the general case, the AIC is AIC=$2k - 2\ln(L)$, where k is the number of parameters in the statistical model, and L is the maximized value of the likelihood function for the estimated model. We believe that although these models are analyzed by AIC, the result will still remain valid if analyzed by other information criterion [16]. The AIC is used only as an information criterion and is not compared with other criterion in this study.

In time series analysis, the Moving-average (MA) model is a common approach for modeling univariate stationary time series. Time series can be expressed to be generated by passing white noises through a non-recursive linear filter. The notation MA(q) refers to the moving average model of order q:

$$X_t = \mu_t + \sum_{k=0}^{q} \theta_k \varepsilon_{t-k},$$

where μ_t is the mean of the series at the time point t, the θ_k are the parameters of the model, and the ε_{t-k} are white noise error terms, $\varepsilon_t \sim N(0,1)$. The value of q is called the order of the MA model. We can assume, without loss of generality, that μ_t is zero at each time point t.

A moving average model is conceptually a linear regression of the current value of the series against previous (unobserved) white noise error terms or random shocks. In stationary time series analysis, the random shocks at each point are assumed to be mutually independent and