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TRANSFORMATIONS FOR THE PRIZE-COLLECTING STEINER TREE PROBLEM AND THE MAXIMUM-WEIGHT CONNECTED SUBGRAPH PROBLEM TO SAP*

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Abstract

Transformations of Steiner tree problem variants have been frequently discussed in the literature. Besides allowing to easily transfer complexity results, they constitute a central pillar of exact state-of-the-art solvers for well-known variants such as the Steiner tree problem in graphs. In this article transformations for both the prize-collecting Steiner tree problem and the maximum-weight connected subgraph problem to the Steiner arborescence problem are introduced for the first time. Furthermore, the considerable implications for practical solving approaches will be demonstrated, including the computation of strong upper and lower bounds.

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1. Introduction

The classical Steiner tree problem in graphs has been investigated for long time, latest during the 11th DIMACS Challenge dedicated to the study of Steiner tree problems. In practical applications, however, the problem usually arises in modified form. As a result, there exist a large number of problem variants. One of the oldest and most widely known is the prize-collecting Steiner tree problem [10], while during the last years the related maximum-weight connected subgraph problem has received considerable attention [2–4].

Many transformations between Steiner problem variants are known, most notably perhaps the one of the Steiner tree problem in graphs to its directed kinsman, the Steiner arborescence problem. This transformation is used by state-of-the-art solvers for the Steiner tree problem in graphs [8,16] for row generation within branch-and-cut [11,16] as well as for powerful reduction techniques [5,16]. Other well-known transformations include those for the rectilinear Steiner tree problem and the group Steiner tree problem to the Steiner tree problem in graphs [6,9].

In order to apply results that have been achieved for the classical Steiner tree problem in graphs it is particularly convenient, if possible, to transform the variant at hand to the classical problem, either in its undirected or its directed form. But while any polynomial transformation

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is enough to transfer complexity results, efficient transformations in the size of the transformed problem can also allow to employ existing advanced solvers for the Steiner tree problem in graphs.

In the following we will present transformations for the (rooted and unrooted) prize-collecting Steiner tree problem and the maximum-weight connected subgraph problem to the Steiner arborescence problem. These novel transformations allow to use state-of-the-art solvers for the Steiner tree problem in graphs to solve the problems. Moreover, we demonstrate how to use the transformations to employ powerful – primal and dual – heuristics to obtain strong upper and lower bounds within very short time.

2. Transforming RPCSTP and PCSTP to SAP

First, we define the *Steiner arborescence problem* (*SAP*), which constitutes the result of all transformations described in this article. Given a directed graph $D = (V, A)$, costs $c : A \to \mathbb{Q}_{\geq 0}$, a set $T \subseteq V$ of terminals, and a root $r \in T$, a directed tree $S = (V_S, A_S) \subseteq D$ is required that first, for all $t \in T$ contains exactly one directed path from r to t and second, minimizes

$$C(S) := \sum_{a \in A_S} c_a. \tag{2.1}$$

Throughout this article the notation $\mathbb{Q}_{\geq 0} := \{x \in \mathbb{Q} \mid x \geq 0\}$ is used.

Having defined the resulting problem, we now turn towards the other end and introduce the *prize-collecting Steiner tree problem* (*PCSTP*), a variant well-studied both theoretically and practically [10, 13]. Given an undirected graph $G = (V, E)$, edge weights $c : E \to \mathbb{Q}_{\geq 0}$, and vertex weights $p : V \to \mathbb{Q}_{\geq 0}$, a tree $S = (V_S, E_S)$ in G is required such that

$$P(S) := \sum_{e \in E_S} c_e + \sum_{v \in V \setminus V_S} p_v \tag{2.2}$$

is minimized. Hereinafter it is assumed for ease of presentation that at least one vertex v is of positive weight (otherwise any vertex constitutes an optimal solution).

To set the stage, we first introduce a transformation for a problem closely related to the PCSTP, the *rooted prize-collecting Steiner tree problem* (*RPCSTP*). This variant incorporates the additional condition that one distinguished node r, called *root*, is part of every feasible solution to the problem.

Transformation 1 (RPCSTP to SAP)

Input: *An RPCSTP* $P = (V, E, c, p, r)$

Output: *An SAP* $P' = (V', A', T', c', r')$

1. *Set* $V' := V$, $A' := \{(v, w) \in V' \times V' \mid \{v, w\} \in E\}$, $r' := r$ *and* $c' : A' \to \mathbb{Q}_{\geq 0}$ *with* $c'_a = c_{\{v,w\}}$ *for* $a = (v, w) \in A'$.
2. *Denote the set of all* $v \in V$ *with* $p_v > 0$ *by* $T = \{t_1, ..., t_s\}$. *For each node* $t_i \in T$, *a new node* t'_i *and an arc* $a = (t_i, t'_i)$ *with* $c'_a = 0$ *is added to* V' *and* A' *respectively.*
3. *Add arcs* (r', t'_i) *for each* $i \in \{1, ..., s\}$, *setting their respective weight to* p_{t_i}.
4. *Define the set of terminals* $T' := \{t'_1, ..., t'_s\} \cup \{r\}$.
5. ***Return*** (V', A', T', c', r').