Journal of Computational Mathematics
Vol.36, No.1, 2018, 1–16.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1605-m2015-0407

A SURVEY OF OPEN CAVITY SCATTERING PROBLEMS*

Peijun Li
Department of Mathematics, Purdue University, West Lafayette, Indiana, 47907, USA
Email: lipeijun@math.purdue.edu

Abstract

This paper gives a brief survey of recent developments on mathematical modeling and analysis of the open cavity scattering problems, which arise in diverse scientific areas and have significant industrial and military applications. The scattering problems are studied for the two-dimensional Helmholtz equation corresponding to the transverse magnetic or electric polarization, and the three-dimensional time-harmonic and time-domain Maxwell equations. Since these problems are imposed in open domains, a key step of the analysis is to develop transparent boundary conditions and reformulate them equivalently into boundary value problems in bounded domains. The well-posedness of weak solutions are shown for the associated variational problems by using either the Lax–Milgram theorem or the Fredholm alternative.

Mathematics subject classification: 35Q61, 78A25, 78M30
Key words: Cavity scattering problem, Helmholtz equation, Maxwell's equations, Transparent boundary condition, Variational problem, Well-posedness.

1. Introduction

The phenomenon of electromagnetic scattering by open cavities has received much attention by many researchers in the engineering and applied mathematics communities. An open cavity is a bounded domain embedded in the ground with its opening aligned with the ground surface. Given the cavity, the direct scattering problem is to determine how the wave is scattered by the cavity. The inverse problem is to answer what information can be extracted about the cavity from the measured wave field. The open cavity scattering problems arise in diverse scientific areas and have significant industrial and military applications. For instance, the radar cross section (RCS) measures the detectability of a target by a radar system. Deliberate control in the form of enhancement or reduction of the RCS of a target is highly important. The cavity RCS caused by jet engine inlet ducts or cavity-backed antennas can dominate the total RCS. A thorough understanding of the electromagnetic scattering characteristic of a target, particularly a cavity, is necessary for successful implementation of any desired control of its RCS.

The time-harmonic problems were introduced and studied firstly by engineers [22–25, 32, 34, 43]. Mathematical analysis of the problems were done in three fundamental papers [2–4], where transparent boundary conditions, based on the Fourier transform, were proposed on the opening. As the applied mathematics community has begun to work on these problems, there has been a rapid development of the theory, analysis, and computational techniques in this area. The mode matching method was developed to find analytic solutions for rectangular cavities [7, 14]. The analytic solutions provide a good understanding of the highly oscillatory

* Received September 29, 2015 / Accepted May 19, 2016 /
Published online October 11, 2017 /

nature of the large cavity problem, which is a perfect example for the long-standing high frequency scattering problems. Tremendous effort was made to develop various fast and accurate numerical methods to solve the large cavity problem [1, 11, 17, 18, 26, 27, 40, 41, 44, 45]. The challenging mathematical issue is to establish the stability estimates with explicit dependence on the high wavenumber [12,13,27], which help us gain a deeper understanding on high frequency problems. To include the analysis for more complex geometries, the overfilled cavity problems were investigated in [16, 31, 42], where the inhomogeneous medium filling the cavity interior was allowed to protrude into the space above the ground surface; the multiple cavity scattering problem was studied in [30], where the cavity was consisted of finitely many disjoint components.

The inverse cavity scattering problem is clearly challenging due to the nonlinearity and lack of stability, i.e., small variations in the data may give rise to large errors in the reconstructions. In the inverse problem community, it seems that more attention is paid on inverse medium, obstacle, or source scattering problems than on the inverse cavity scattering problem. Hence it is less studied. The results on uniqueness and local stability may be found in [6, 19, 28, 33]. Related optimal design problems can be found in [8–10], which was to design the shape of the cavity so as to minimize the RCS. We also refer to [5, 15] for the study on the electromagnetic field enhancement by interacting subwavelength cavities.

The time-domain electromagnetic scattering problems have attracted much attention due to their capability of capturing wide-band signals and modeling more general material and nonlinearity. Comparing with the time-harmonic problems, the time-domain problems are also less studied due to the additional challenge of the temporal dependence. The transient cavity scattering problems were examined in [20, 21, 35–39], where the focus was on temporal discretization and the analysis of the finite element method. A theoretical analysis can be found in [29] for the transient electromagnetic scattering from a three-dimensional open cavity.

The goal of this paper is to give a brief survey of recent developments on mathematical modeling and analysis of the open cavity scattering problems. Particular emphasis is on the formulation of the mathematical models, which include the two-dimensional Helmholtz equation corresponding to the transverse magnetic and electric polarizations and the three-dimensional time-harmonic and time-domain Maxwell equations. Since the problems are imposed in open domains, a key step of the analysis is to develop transparent boundary conditions and reformulate them equivalently into boundary value problems in bounded domains. The well-posedness of the weak solutions are presented for the associated variational problems by using either the Lax–Milgram theorem or the Fredholm alternative.

The paper is outlined as follows. In Section 2, the two-dimensional Helmholtz equations are introduced for the two fundamental polarizations. Section 3 and 4 are concerned with the three-dimensional time-harmonic and time-domain Maxwell equations, respectively. Topics are organized to present model problems, transparent boundary conditions, and well-posedness of weak solutions corresponding to each of these three sections. The paper is concluded with some general remarks and directions for future research in Section 5.

2. The Helmholtz Equation

We begin with a simpler model for the open cavity scattering problem and consider the two-dimensional Helmholtz equation by assuming that the structure is invariant along the z-axis.