Journal of Computational Mathematics
Vol.35, No.1, 2017, 91–120.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1601-m2014-0175

BASES OF BIQUADRATIC POLYNOMIAL SPLINE SPACES OVER HIERARCHICAL T-MESHES*

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Abstract

Basis functions of biquadratic polynomial spline spaces over hierarchical T-meshes are constructed. The basis functions are all tensor-product B-splines, which are linearly independent, nonnegative and complete. To make basis functions more efficient for geometric modeling, we also give out a new basis with the property of unit partition. Two preliminary applications are given to demonstrate that the new basis is efficient.

Mathematics subject classification: 65D07
Key words: Spline spaces over T-meshes, CVR graph, Basis functions.

1. Introduction

NURBS is a basic tool in surface modeling and isogeometric analysis (IGA). However, NURBS suffer from the weakness that the control points must lie topologically in a tensor-product mesh. If we want to construct a surface which is flat in the most part of the domain, but sharp in a small region, we have to use more control points not only in the sharp region, but also in the flat region to maintain the tensor-product structure. The superfluous control points are a burden. To overcome this limitation, we need splines which can be refined locally. This type of splines are defined on T-meshes which allow T-junctions.

Hierarchical B-splines were first introduced in [13], defined on a nest sequence of hierarchical meshes that can be locally refined, maintain a local tensor product structure and rely on the principle of B-spline subdivision. The hierarchical structure and the linear independence of hierarchical B-splines make it appealing for isogeometric analysis [15, 28]. For the partition of unity, THB-splines were introduced in [14]. However, the completeness of hierarchical B-splines can not be guaranteed generally.

In 2003, T-splines [20] were proposed. T-splines have been widely used in geometric modeling [21, 22] and IGA [1, 2, 5, 24]. However, the use of the T-splines in analysis has exhibited a number of problems that have been addressed by the researchers, such as the linear dependence of blending functions [4]. Analysis-suitable T-splines were introduced in [16, 17, 23] to deal with the problem. The linear independence of analysis-suitable T-splines makes it suitable for

* Received January 6, 2015 / Revised version received December 22, 2015 / Accepted January 14, 2016 / Published online January 18, 2017 /

analysis. However, analysis-suitable T-splines are defined on the analysis-suitable T-meshes. We need to refine more cells to convert a T-mesh to an analysis-suitable T-mesh.

LR B-splines [9] as a collection of hierarchically scaled B-splines are defined on a special type of T-meshes (LR-meshes). LR B-splines are not always linearly independent although there exists an algorithm to check whether the spline functions are linearly independent and convert the spline functions to be linearly independent by inserting more control points.

The splines mentioned above are not defined from the viewpoint of space. Therefore, the linear independence and completeness are two challenges. In 2006, spline spaces over T-meshes [6] were put forward by one of the present authors. This type of splines is defined directly from the viewpoint of spline spaces, dimension formula and basis functions are the main problems. In [6], each function in $\mathbf{S}(m,n,\alpha,\beta,\mathscr{T})$ over a T-mesh $\mathscr{T}$ is a bi-degree (m,n) polynomial in each cell of $\mathscr{T}$ with smoothness order α and β along two directions. A dimension formula is given under the condition of $m \geq 2\alpha+1$ and $n \geq 2\beta+1$. In 2008, polynomial splines over hierarchical T-meshes (PHT-splines) [7], which usually refer to bicubic C^1 polynomial splines over hierarchical T-meshes, were proposed. PHT-splines have been used efficiently and adaptively in isogeometric analysis [25–27, 31] and surface modeling [18, 32]. However, excessive growth of dimension is a challenge in the application of PHT-splines, especially in three-dimensional space. To deal with this problem, we consider the spline spaces $\mathbf{S}(m,n,m-1,n-1,\mathscr{T})$. Unfortunately, [3, 19] observed that the dimension of $\mathbf{S}(m,n,m-1,n-1,\mathscr{T})$ may depend on the geometry of the T-mesh. Thus, more attention is paid to some special classes of T-meshes.

It is noted that [29] gave a general dimension formula of $\mathbf{S}(m,n,m-1,n-1,\mathscr{T})$ over a special T-mesh ((m,n)-subdivided T-mesh). Hierarchical bases of $\mathbf{S}(m,n,m-1,n-1,\mathscr{T})$ over (m,n)-subdivided T-mesh were constructed in [30]. However, hierarchical bases in [30] did not have the properties of non-negativity and partition of unity. [34] gave a dimension formula of $\mathbf{S}(3,3,2,2,\mathscr{T})$ over a more general T-mesh than that in [29], but also a special hierarchical T-mesh. A basis of $\mathbf{S}(3,3,2,2,\mathscr{T})$ was given in [33] under three rules of refinement of the hierarchical T-mesh.

In this paper, we give a basis of $\mathbf{S}(2,2,1,1,\mathscr{T})$ based on the topological explanation of dimension formula in [8]. The basis functions are all tensor-product B-splines, which are linearly independent, nonnegative and complete. As unit partition is an important property for geometry modeling, a new basis with the property of unit partition is given out by an elementary transformation of the previous basis functions. Compared with spline bases in [30], the new basis has good properties of nonnegativity, partition of unity, and smaller local support which may result in a more stable stiffness matrix in numerical solutions of PDE. Also, these properties will facilitate their applications in geometric modeling. In fact, when representing a geometric shape with a linear combination of basis functions satisfying these properties, we can implement local shape control, affine invariance, and variational diminishing. See [11] for details. In geometric modeling, spline functions with C^1 are necessary in some situations.

The remainder of this paper is organized as follows. In Section 2, the spline spaces over T-meshes are reviewed, and several results from paper [8] are listed. In Section 3, the rule of refinement is elaborated. Construction of basis functions is discussed in Section 4. Section 5 gives the proof of the linear independence of the basis. In Section 6, a new basis with unit partition is given. In Section 7, we will present two preliminary applications of the new basis. We end the paper with conclusions and future work in Section 8.