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A PRIMAL-DUAL FIXED POINT ALGORITHM FOR MULTI-BLOCK CONVEX MINIMIZATION*

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Abstract

We have proposed a primal-dual fixed point algorithm (PDFP) for solving minimization of the sum of three convex separable functions, which involves a smooth function with Lipschitz continuous gradient, a linear composite nonsmooth function, and a nonsmooth function. Compared with similar works, the parameters in PDFP are easier to choose and are allowed in a relatively larger range. We will extend PDFP to solve two kinds of separable multi-block minimization problems, arising in signal processing and imaging science. This work shows the flexibility of applying PDFP algorithm to multi-block problems and illustrates how practical and fully splitting schemes can be derived, especially for parallel implementation of large scale problems. The connections and comparisons to the alternating direction method of multiplier (ADMM) are also present. We demonstrate how different algorithms can be obtained by splitting the problems in different ways through the classic example of sparsity regularized least square model with constraint. In particular, for a class of linearly constrained problems, which are of great interest in the context of multi-block ADMM, can be also solved by PDFP with a guarantee of convergence. Finally, some experiments are provided to illustrate the performance of several schemes derived by the PDFP algorithm.

Mathematics subject classification: 65K05, 46N10, 90C06, 90C25
Key words: Primal-dual fixed point algorithm, Multi-block optimization problems.

1. Introduction

In this paper, we are concerned with extending the primal-dual fixed point (PDFP) algorithm proposed in [5] for solving two kinds of general multi-block problems (1.1) and (1.2) with fully splitting schemes. Let $\Gamma_0(\mathbb{R}^n)$ denote the collection of all proper lower semicontinuous convex functions from $\mathbb{R}^n$ to $(-\infty, +\infty]$. The first kind of problems are formulated as

$$\min_{x\in\mathbb{R}^n} f_1(x) + \sum_{i=1}^{N} \theta_i(B_i x + b_i) + f_3(x), \tag{1.1}$$

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where $\theta_i \in \Gamma_0(\mathbb{R}^{m_i})$, $B_i : \mathbb{R}^n \to \mathbb{R}^{m_i}$ a linear transform, $b_i \in \mathbb{R}^{m_i}$, $i = 1, \cdots, N$. $f_1, f_3 \in \Gamma_0(\mathbb{R}^n)$ and f_1 is differentiable on $\mathbb{R}^n$ with $1/\beta$-Lipschitz continuous gradient for some $\beta \in (0, +\infty]$. If $f_1 = 0$, we can take $\beta = +\infty$. Many problems in image processing and signal recovery with multi-regularization terms can be formulated in the form of (1.1).

The second kind of problems under discussion are optimization problems with constraints, given as follows.

$$\min_{x_1,\cdots,x_N} \sum_{i=1}^{N_1} \theta_i(B_i x_i + b_i) + \sum_{i=N_1+1}^{N} \theta_i(x_i) \tag{1.2a}$$

$$\text{st. } \sum_{i=1}^{N} A_i x_i = a, \tag{1.2b}$$

$$x_i \in C_i, \quad i = 1, \cdots, N.$$

Here, $\theta_i \in \Gamma_0(\mathbb{R}^{m_i})$, $B_i : \mathbb{R}^{n_i} \to \mathbb{R}^{m_i}$ a linear transform and $b_i \in \mathbb{R}^{m_i}$ for $i = 1, \cdots, N_1$. Moreover, for $i = N_1 + 1, \cdots, N$, $\theta_i \in \Gamma_0(\mathbb{R}^{n_i})$ is differentiable on $\mathbb{R}^{n_i}$ with $1/\beta_i$-Lipschitz continuous gradient for some $\beta_i \in (0, +\infty]$. For $i = 1, \cdots, N$, the constraint set $C_i \subset \mathbb{R}^{n_i}$ is nonempty, closed and convex, A_i is a $l \times n_i$ matrix, and $a \in \mathbb{R}^l$.

Many problems can be formulated in the form (1.2), for example elliptic optimal control problems [6]. In some applications, the problem (1.1) can be viewed as a decomposition on the observed data, while the problem (1.2) is a mixture of the variables and data decomposition. In particular, for some special cases, both problems (1.1) and (1.2) can be abstracted as

$$\min_{x_1,x_2,\cdots,x_N} \sum_{i=1}^{N} \theta_i(x_i) \tag{1.3a}$$

$$\text{st. } \sum_{i=1}^{N} A_i x_i = a, \tag{1.3b}$$

$$x_i \in C_i, \quad i = 1, \cdots, N,$$

by properly introducing auxiliary variables, or vice-visa, depending on the simplicity of the functions θ_i involved. In the literature, many existing works have been devoted to solving (1.3), for example, the variants of popular alternating direction method of multipliers (ADMM) [9,11, 12] for three or more block problems. It deserves to point out that Davis and Yin [10] proposed a very interesting "primal-only" splitting scheme for solving an inclusion problem involving three maximal monotone operators, which was also used for solving (1.3) by themselves.

Now, let us recall the primal-dual fixed point algorithm PDFP in [5] for solving the following three-block problem

$$\min_{x \in \mathbb{R}^n} f_1(x) + f_2(Bx + b) + f_3(x). \tag{1.4}$$

In (1.4), $f_2 \in \Gamma_0(\mathbb{R}^m)$, $B : \mathbb{R}^n \to \mathbb{R}^m$ a linear transform, $b \in \mathbb{R}^m$, f_1 and f_3 are the same ones as given in (1.1). As usual, define the proximity operator prox_f of f by (cf. [7])

$$\text{prox}_f(x) = \arg\min_{y \in \mathbb{R}^n} f(y) + \frac{1}{2}\|x - y\|^2.$$