Journal of Computational Mathematics
Vol.34, No.1, 2016, 1–11.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1506-m2014-0110

# STRONG PREDICTOR-CORRECTOR METHODS FOR STOCHASTIC PANTOGRAPH EQUATIONS*

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**Abstract**

The paper introduces a new class of numerical schemes for the approximate solutions of stochastic pantograph equations. As an effective technique to implement implicit stochastic methods, strong predictor-corrector methods (PCMs) are designed to handle scenario simulation of solutions of stochastic pantograph equations. It is proved that the PCMs are strong convergent with order $\frac{1}{2}$. Linear MS-stability of stochastic pantograph equations and the PCMs are researched in the paper. Sufficient conditions of MS-unstability of s-tochastic pantograph equations and MS-stability of the PCMs are obtained, respectively. Numerical experiments demonstrate these theoretical results.

*Mathematics subject classification:* 60H10, 65C20.
*Key words:* Stochastic pantograph equation, Predictor-corrector method, MS-convergence, MS-stability.

## 1. Introduction

In 1971, Ockendon and Tayler [15] used the equation $\dot{x(t)} = ax(t) + bx(pt)$ to model the collection of current by pantograph of an electric locomotive. This is the origin of the 'pantograph' in 'pantograph differential equations'. From then on, pantograph differential equations arise widely in dynamical systems, probability, quantum mechanics, electrodynamics and so on. A wealth of literature exists on analytical solution as well as numerical solution. The early related results can be found in [5, 8, 10], and the referents therein. More recently results can be found in [2, 4, 9, 12, 13].

Stochastic pantograph equation can be viewed as a generalization of the deterministic pantograph differential equation which takes into account of random factors. It possesses a wide range of applications. Up to now, only few results of stochastic pantograph equation have been presented. In 2000, Baker and Buckwar [1] obtained the necessary analytical theory for existence and uniqueness of strong approximations of a continuous extension of the $\theta$-Euler methods and established 1/2 mean-square convergence of approximations. In 2007, Fan, Liu and Cao [7] discussed the existence and uniqueness of solutions and convergence of semi-implicit euler methods for stochastic pantograph equation. Some criteria for linear asymptotically mean square stability was given in [6]. In 2009, Xiao and Zhang [17] proved $\theta$-methods of nonlinear stochastic pantograph equation are MS-stabile under appropriate conditions. In 2011, Xiao

* Received October 24, 2014 / Revised version received May 11, 2015 / Accepted June 11, 2015 / Published online January 18, 2016 /

and Zhang [19]constructed numerical methods with variable step size to solve stochastic pantograph equation, the convergence and linear MS-stability were discussed. In 2013, Xiao,Zhang and Qin [18] discusses the MS-stability of the milstein method for stochastic pantograph equations.

For deterministic ordinary differential equations, the numerical stability of explicit methods are generally worse than implicit methods and the disadvantage of implicit methods is that an algebraic equation needs to be solved at each time step. It has been well known that PCMs can improve numerical stability comparing with standard explicit methods and don't require to solve an algebraic equations. For SDEs, the PCMs have the same properties. Weak PCMs for SDEs were discussed in [16] and [11]. In [3], a family of strong predictor-corrector Euler methods is designed to simulate the solution of SDEs. In [14], Niu and Zhang established a class of PCMs for SDEs and proved that the PCMs maintain almost sure and moment exponential stability for sufficiently small timesteps.As far as we know, it doesn't exist any literature about PCMs for SDEs with delay (SDDEs).

In this article, we deal with stochastic pantograph equation

$$\begin{cases} dx(t) = f(x(t), \quad x(pt))dt + g(x(t), \quad x(pt))dw(t), \quad t_0 < t, \quad p \in (0,1); \\ x(t) \quad = \xi(t), \quad pt_0 \le t \le t_0. \end{cases} \tag{1.1}$$

The work is organized as follows: Section 2 analyzes mean-square bound and stability of stochastic pantograph differential equations and establishes a family of predictor-corrector methods (PCMs $(\theta, \eta)$) to simulate approximation of the stochastic pantograph differential equations. In Section 3, the convergence is discussed. It proved that the PCMs $(\theta, \eta)$ is mean square numerical convergent with order 1/2. In Section 4, some linear numerical MS-stability criteria of PCMs $(\theta, \eta)$ are obtained. If stochastic pantograph differential equations are MS-stable, then the numerical solutions of PCMs $(\theta, \eta)$ are MS-stable under appropriate conditions. Section 5 gives some numerical experiments to illustrate the obtained theoretical results.

## 2. Predictor-corrector Methods for Stochastic Pantograph Differential Equations

Let $(\Omega, \mathcal{A}, P)$ be a complete probability space with a filtration $(\mathcal{A}_t)_{t \ge t_0}$, which is right-continuous and satisfies that each $\mathcal{A}_t$ $(t \ge t_0)$ contains all $P$-null sets in $\mathcal{A}$, and $w$ is an d-dimensional Brownian motion defined on the probability space, $|\cdot|$ is the trace norm, $E_t(\cdot) = E(\cdot|\mathcal{A}_t)$.

We integrate(1.1) and obtain

$$\begin{cases} x(t) = x(t_0) + \int_{t_0}^{t} f(x(s), \quad x(ps))ds + \int_{t_0}^{t} g(x(s), \quad x(ps))dw(s), \quad t_0 \le t \le T, \\ x(t) = \xi(t), \quad pt_0 \le t \le t_0, \end{cases} \tag{2.1}$$

where $x(t)$ is a $R^d$-value random process, $p \in (0,1)$ denotes a given constant, the second integral is Itô type, $f : R^d \times R^d \to R^d$ and $g : R^d \times R^d \to R^d$ are two given Borel-measurable functions, $\xi(t)$ is a $C([pt_0, t_0], R^d)$-value initial segment with $E\|\xi(t)\| < \infty$, let $\|\xi\| = \sup_{pt_0 \le t \le t_0} |\xi(t)|$. Throughout this paper, we assume the Eq. (2.1) has an uniqueness solution $x(t) \in \mathcal{M}^2(R_+, R^d)$ and satisfies the following conditions:

(C1) (global Lipschitz condition )

$$|f(x,u) - f(y,v,)|^2 \vee |g(x,u) - g(y,v)|^2 \le \beta_1(|x-y|^2 + |u-v|^2); \tag{2.2}$$