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A POSTERIORI ERROR ANALYSIS OF A FULLY-MIXED FINITE ELEMENT METHOD FOR A TWO-DIMENSIONAL FLUID-SOLID INTERACTION PROBLEM *

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Abstract

In this paper we develop an a posteriori error analysis of a fully-mixed finite element method for a fluid-solid interaction problem in 2D. The media are governed by the elastodynamic and acoustic equations in time-harmonic regime, respectively, the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements, and the fluid is supposed to occupy an annular region surrounding the solid, so that a Robin boundary condition imitating the behavior of the Sommerfeld condition is imposed on its exterior boundary. Dual-mixed approaches are applied in both domains, and the governing equations are employed to eliminate the displacement $\mathbf{u}$ of the solid and the pressure p of the fluid. In addition, since both transmission conditions become essential, they are enforced weakly by means of two suitable Lagrange multipliers. The unknowns of the solid and the fluid are then approximated by a conforming Galerkin scheme defined in terms of PEERS elements in the solid, Raviart-Thomas of lowest order in the fluid, and continuous piecewise linear functions on the boundary. As the main contribution of this work, we derive a reliable and efficient residual-based a posteriori error estimator for the aforedescribed coupled problem. Some numerical results confirming the properties of the estimator are also reported.

Mathematics subject classification: 65N30, 65N15, 74F10, 74B05, 35J05.
Key words: Mixed finite elements, Helmholtz equation, Elastodynamic equation, A posteriori error analysis.

1. Introduction

In the recent paper [14] we introduced and analyzed a fully-mixed finite element method for the two-dimensional fluid-solid interaction problem studied originally in [17] (see also [18]). The respective model consists of an elastic body which is subject to a given incident wave that travels in the fluid surrounding it. Actually, the fluid is supposed to occupy an annular region,

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and hence a Robin boundary condition imitating the behavior of the scattered field at infinity is imposed on its exterior boundary, which is located far from the obstacle. The media are governed by the elastodynamic and acoustic equations in time-harmonic regime, respectively, and the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements. Differently from the analysis in [17] where dual and primal methods are utilized in the solid and fluid, respectively, dual-mixed approaches are applied in both domains in [14], and the governing equations are employed to eliminate the displacement $\mathbf{u}$ of the solid and the pressure p of the fluid. In addition, since both transmission conditions become essential, they are enforced weakly by means of two suitable Lagrange multipliers. In this way, the Cauchy stress tensor and the rotation of the solid, together with the gradient of p and the traces of $\mathbf{u}$ and p on the boundary of the fluid, constitute the unknowns of the coupled problem. The solvability of the resulting continuous formulation is analyzed in [14] by incorporating first suitable decompositions of the spaces to which the stress and the gradient of p belong, and then by applying the Babuška-Brezzi theory and the Fredholm alternative. The unknowns of the solid and the fluid are approximated by a conforming Galerkin scheme defined in terms of PEERS elements in the solid, Raviart-Thomas of lowest order in the fluid, and continuous piecewise linear functions on the boundary. The analysis of the discrete method relies on a stable decomposition of the corresponding finite element spaces and also on the classical result on projection methods for Fredholm operators of index zero.

On the other hand, it is well known that in order to guarantee a good convergence behaviour of the finite element solutions, specially under the presence of complex geometries leading eventually to singularities, one needs to apply an adaptive strategy based on a posteriori error estimates. These are usually represented by global quantities $\boldsymbol{\theta}$ that are expressed in terms of local estimators θ_T defined on each element T of a given triangulation of the domain. The estimator $\boldsymbol{\theta}$ is said to be reliable (resp. efficient) if there exists $C_{\texttt{rel}} > 0$ (resp. $C_{\texttt{eff}} > 0$), independent of the meshsizes, such that

$$C_{\texttt{eff}}\, \boldsymbol{\theta} + \text{h.o.t.} \;\leq\; \|error\| \;\leq\; C_{\texttt{rel}}\, \boldsymbol{\theta} + \text{h.o.t.}\,,$$

where h.o.t. is a generic expression denoting one or several terms of higher order. Concerning the Helmholtz and elasticity equations, several approaches have already been developed independently in the literature. In particular, a posteriori error analyses for interior Helmholtz problems, which are based on local computations or explicit residuals, can be found in [7] and [24], respectively. In addition, a reliable residual-based a posteriori error estimator, which follows the nowadays standard approach from [29], is proposed in [25]. In turn, a posteriori error estimators for the mixed finite element formulation of the linear elasticity problem, which are based on residuals and on the solution of local problems, are provided in [2]. The main novelty of the approach there has to do with the utilization of a Helmholtz decomposition of the stress-type unknown to derive the corresponding reliability and efficiency estimates. For related approaches employing the Helmholtz decomposition technique as well we refer to [11] and [26].

Furtermore, to the best of our knowledge, [16] is the only work available in the literature dealing with the a posteriori error analysis of fluid-solid interaction problems involving the acoustic and elastodynamic equations in time-harmonic regime. In fact, a reliable and efficient residual-based a posteriori error estimator for the dual-mixed/primal formulation of the model problem analyzed in [17] was derived in [16]. More precisely, suitable auxiliary problems, the continuous inf-sup conditions satisfied by the bilinear forms involved, a discrete Helmholtz decomposition,