Journal of Computational Mathematics
Vol.33, No.4, 2015, 341–355.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1412-m4494

SPECTRAL DY-TYPE PROJECTION METHOD FOR NONLINEAR MONOTONE SYSTEM OF EQUATIONS*

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Abstract

In this paper, we propose a spectral DY-type projection method for nonlinear monotone system of equations, which is a reasonable combination of DY conjugate gradient method, the spectral gradient method and the projection technique. Without the differentiability assumption on the system of equations, we establish the global convergence of the proposed method, which does not rely on any merit function. Furthermore, this method is derivative-free and so is very suitable to solve large-scale nonlinear monotone systems. The preliminary numerical results show the feasibility and effectiveness of the proposed method.

Mathematics subject classification: 65F10, 65K05.
Key words: Nonlinear monotone system of equations, spectral gradient method, DY conjugate gradient method, Projection method, Global convergence.

1. Introduction

In this paper, we consider the following nonlinear monotone system of equations

$$F(x) = 0, \tag{1.1}$$

where F: $R^n \to R^n$ is continuous and monotone, i.e.

$$(F(x) - F(y))^T (x - y) \geq 0, \quad \forall x, y \in R^n.$$

It is not difficult to find that the solution set of the problem (1.1) is convex.

Nonlinear monotone system of equations generally arise in various applications such as ballistic trajectory commutation and vibration systems [1, 2], the first-order necessary conation of the unconstrained convex optimization problem and subproblems of the generalized proximal algorithm [3]. Some monotone variational inequality also can be converted into nonlinear monotone system of equations by means of the fixed point map or normal map [4]. Consequently, a variety of different iterative methods have been developed for solving the problem (1.1), for example, Newton method, quasi-Newton method, Gauss-Newton methods, Levenberg-Marquardt method and their variants. These methods are attractive because they converge rapidly from

* Received February 22, 2014 / Revised version received October 29, 2014 / Accepted December 13, 2014 / Published online July 17, 2015 /

a good initial guess (see, e.g., [1, 5, 6]). However, they are not particularly suitable for solving large-scale nonlinear monotone system of equations because they need to solve a linear system of equations using a Jacobian matrix or an approximation Jacobian matrix at each iteration.

It is well-know that the spectral gradient method is very suitable to solve large-scale optimization problems and nonlinear system of equations (see e.g., [7-11]). The first spectral gradient method was originally proposed by Barzilai and Borwein [7] for unconstrained optimization problems. Raydan [8] further studied the Barzilain and Borwein gradient method. An attractive property of this method is that it only needs gradient directions at each line search whereas a non-monotone strategy guarantees the global convergence. It was also extended to solve large-scale nonlinear system of equations by Cruz and Raydan [10]. Recently, Cruze et al [11] introduced a new nonmonotone linear search technique for the spectral gradient method to solve nonlinear system of equations. Zhang and Zhou [12] combine the spectral gradient method [7] with the projection technique [13] to obtain a spectral gradient projection method for solving nonlinear monotone system of equations. The preliminary numerical results showed that their method was very effective by comparing with the the spectral approach for nonlinear system of equations (SANE)in [10] and Inexact Newton Method (INM) in [13]. In [12], if F is a gradient vector of a real valued function $f : R^n \to R$, the spectral gradient projection method only uses the negative gradient vector as a search direction. This may make the iterate points zigzag, which is similar with the disadvantage of the steepest decent method [14]. Many efforts have been devoted to overcoming this drawback of the steepest decent method. One of successful examples is to establish the conjugate gradient method, which is very important method for solving the unconstrained optimization problems and nonlinear system of equations. The search direction of the conjugate gradient method is a combination of the negative gradient and the last search direction. Moreover, this technique has been used in the projection and contraction methods for solving variational inequality problems; For detail discussions, please see the reference [15].

Inspired by the structure of the search direction in the conjugate gradient method, in this paper we further study the spectral gradient projection method based on DY conjugate gradient method [16] and the projection technique [13] for large-scale nonlinear monotone system of equations. The main difference between our approach and the proposed method in [12] is that the search direction in our approach can be viewed as a combination of the negative gradient and the search direction used in the last iteration. We do not need the differentiability assumption on the system of equations. Under appropriate conditions, the global convergence of the proposed method can be established, which does not rely on any merit function. Simultaneously, the preliminary numerical results also show that our method is robust and effective for solving large-scale nonlinear monotone system of equations.

The paper is organized as follows. In the next section, we firstly recall DY conjugate gradient method, the spectral gradient method, and the projection technique. Then, we introduce our algorithm and its some properties. In Section 3, we give some important lemmas, and establish the global convergence of the proposed method. Preliminary numerical results are presented in Section 4. Finally, we have a conclusion.

Throughout the paper, we use $||\cdot||$ to denote the Euclidean norm of vectors. And the following assumption always holds.

Assumption 1.1. The function $F : R^n \longrightarrow R^n$ is monotone and Lipschiz continuous, that is,