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VARIATIONAL ITERATIVE ALGORITHMS IN PHOTOACOUSTIC TOMOGRAPHY WITH VARIABLE SOUND SPEED*

Tangjie Lv Tie Zhou
LMAM, School of Mathematical Sciences, Peking University, Beijing 100871, China
Email: lvtangjie@gmail.com tzhou@math.pku.edu.cn

Abstract

In this paper, we investigate the adjoint equation in photoacoustic tomography with variable sound speed, and propose three variational iterative algorithms. The basic idea of these algorithms is to compute the original equation and the adjoint equation iteratively. We present numerical examples and show the well performance of these variational iterative algorithms.

Mathematics subject classification: 35L05, 35R30, 65R32, 92C55.
Key words: Photoacoustic Tomography, Variational iterative algorithms.

1. Introduction

Photoacoustic Tomography or Thermoacoustic Tomography is a developing medical imaging method in recent decades [24]. They are hybrid medical imaging methods characterized by high resolution and contrast. The physical principle can be described as follows. Light or a short electromagnetic energy irradiates the biological tissue and the energy is absorbed by the tissue. The tissue heats up and results in the phenomenon of the thermal expansion. This expansion leads to weak acoustic waves and these waves are measured by ultrasound transducers located on an observation surface. The measured information is used to recover the initial acoustic pressure, which is roughly proportional to the rate of absorption [24]. Then the initial acoustic pressure is used to produce an image.

We describe the widely accepted mathematical model here [12, 17]. Let $\Omega \subset \mathcal{R}^n$ be an open set with a smooth boundary, in applications, $n = 2, 3$. Assume that the sound speed $c(x) \in C^\infty(\mathcal{R}^n)$ is smooth, strictly positive and $c(x) = 1$ outside Ω. Suppose $u(x,t)$ is the solution of the wave equation

$$u_{tt} - c^2 \Delta u = 0, \quad t \geq 0,\ x \in \mathcal{R}^n, \tag{1.1a}$$

$$u|_{t=0} = f, \tag{1.1b}$$

$$u_t|_{t=0} = 0, \tag{1.1c}$$

in which the function f is supported in Ω and the value of u could be collected at transducer's location $x \in S$ before a fixed measuring time T, where $S \subset \partial\Omega$ is the closed observation surface. The measured value could be modeled by an operator as follows

$$\mathbf{K} f := u|_{S\times[0,T]}. \tag{1.2}$$

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The image reconstruction problem in photoacoustic tomography is to recover the initial value $f(x)$ by using the measured value $\mathbf{K}f$.

Obviously, this is an inverse problem. A lot of work have been done when the sound speed $c(x)$ is constant, and there are various types of reconstruction methods to solve this problem (e.g filtered backprojection formulas, eigenfunction expansions and time reversal). Actually, when $n = 3$ and the observation surface is spherical surface, the first inversion formula has been found in [6]. Then in [23], a universal inversion formula has been proposed in the case of the observation surface being spherical, planar and cylinadrical surface. The eigenfunction expansion methods can work for any closed observation surface with constant sound speed and the algorithm reconstructs the image faster than filtered backprojection formulas. However, all of them would not be efficient for variable sound speed [2, 13, 14]. There are many cases that the sound speed is variable [25]. If we apply the reconstruction methods for constant speed on the variable sound speed condition, where we use the average sound speed as the constant sound speed, it may cause large image errors [9]. Therefore, designing an effecient algorithm for photoacoustic tomography with variable sound speed is an important work.

Several image reconstruction methods have been proposed to compensate for weak sound speed variations. These methods assume that the photoacoustic wavefields propagate along well-defined geometrical acoustic rays [16, 25]. However, these models possess limitations. The ray-based propagation models will be effective only on length scales that are large compared to the effective acoustic wavelength. These assumptions can be voilated in preclinical and clinical applications [10].

For strong sound speed variations, the time reversal method has been proposed and it works well in numerical experiments when the measuring time T is large, though it gives the exact reconstruction result only when the sound speed is constant and the dimension is odd [4,8,9,26]. If these conditions are not simultaneously satisfied, it gives the approximate results [8, 9] and the errors will increase when T becomes small [8, 17]. The Neumann series method is derived in [20, 21], in which it is proved to be an exact reconstruction method under the variable sound speed circumstance when T is not large. This method has been validated effective in numerical experiments [17]. In [3], the authors study the adjoint operator of the appoximate photoacoustic tomography model with constant sound speed and propose a conjugate gradient method. They use this method with variable sound speed in numerical experiments. When T is fixed, the conjugate gradient method performs better than time reversal, but worse than Neumann series [3].

In this paper, we investigate the adjoint operator of the exact photoacoustic tomography model with variable sound speed, and propose three variational iterative algorithms. We present numerical examples and show that the proposed algorithms performs better than time reversal and Neumann series, especially with noisy data.

This paper is organized as follows. In Section 2, we introduce some basic theories of the hyperbolic equation. The adjoint of the imaging operator and the adjoint equation are studied in Section 3. In Section 4, three variational iterative algorithms based on the adjoint operator are proposed. Finally, numerical results are presented in Section 5, showing that our proposed approaches reconstruct better results than those obtained by the existing methods.