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GENERALIZED CONJUGATE A-ORTHOGONAL RESIDUAL SQUARED METHOD FOR COMPLEX NON-HERMITIAN LINEAR SYSTEMS*

Jianhua Zhang
Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
Department of Mathematics, Anhui Science and Technology University, Fengyang 233100, China
Email: zhangjhnuaa@gmail.com
Hua Dai
Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
Email: hdai@nuaa.edu.cn

Abstract

Recently numerous numerical experiments on realistic calculation have shown that the conjugate A-orthogonal residual squared (CORS) method is often competitive with other popular methods. However, the CORS method, like the CGS method, shows irregular convergence, especially appears large intermediate residual norm, which may lead to worse approximate solutions and slower convergence rate. In this paper, we present a new product-type method for solving complex non-Hermitian linear systems based on the biconjugate A-orthogonal residual (BiCOR) method, where one of the polynomials is a BiCOR polynomial, and the other is a BiCOR polynomial with the same degree corresponding to different initial residual. Numerical examples are given to illustrate the effectiveness of the proposed method.

Mathematics subject classification: 65F10.
Key words: Krylov subspace, BiCOR method, CORS method, Complex non-Hermitian linear systems.

1. Introduction

Some science and engineering applications, for instance in discretizing Helmhlotz and Maxwell equations, require the solution of large linear systems

$$Ax = b, \tag{1.1}$$

where A is an $N \times N$ complex non-Hermitian matrix and N is large.

In recent years, there have been many advances in Krylov subspace methods for solution of complex non-Hermitian linear systems, see, e.g., [1]. If storage requirement is not considered, the generalized minimal residual (GMRES) method [2] and its variant flexible GMRES (FGMRES) [3] are popular options due to their robustness and smooth convergence, see [4]. In terms of cheaper memory demanding, some of the short-recurrence methods based on Bi-Lanczos process are effective and competitive. The archetype of this class is the BiCG [5] method proposed by

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Fletcher. However, the BiCG method requires the transpose of matrix, suffers from breakdown and converges irregularly. In order to avoid the transpose of matrix, Sonneveld developed the CGS [6] method by making use of the "wasted" extra matrix-vector multiplication. To overcome the erratic residual norms of the CGS method, van der Vorst derived the BiCGSTAB [7] method by incorporating linear minimal residual step at each iteration. Gutknecht, Sleijpen and Fokkema generalized this method to the BiCGSTAB2 [8] and BiCGSTAB(l) [9] methods, respectively. Zhang proposed the generalized product-type of the BiCG (GPBiCG) [10] method and provided a way to show the CGS, BiCGSTAB, BiCGSTAB2 and GPBiCG methods fit into a more general framework. ML(k)BiCGSTAB method [11] is also a BiCGSTAB variant based on multiple left Lanczos starting vectors. In [12], Freund considered an alternative approach and devised the TFQMR method by combining the CGS idea with quasi-minimal residual technique proposed in the QMR [13] method. However, they require many more iterations for some realistic problems [14] compared with the GMRES method.

Recently, Jing et al. [15,16] developed the BiCOR, CORS and BiCORSTAB methods for solving complex non-Hermitian linear system based on the biconjugate A-orthonormalization process, which are also considered as Lanczos-type variants of the conjugate A-orthogonal conjugate residual (COCR) method [17]. Note that an implementation of BiCOR-type methods can be constructed from any BiCG-type method by a formal B-inner product $\langle \widetilde{y}, y\rangle_B = \langle \widetilde{y}, By\rangle$ instead of the standard Hermitian inner product $\langle \widetilde{y}, y\rangle$. The choice of $B = A$ can lead to the BiCOR-type methods, see [18] for more details. As observed from different numerical experiments on some practical physical problems, including radar cross section (RCS) calculation from complex structures, acoustics problems, quantum mechanics and so on, these methods show competitive convergence behavior and are often superior to other Krylov subspaces, see [19-21] for details. In order to accelerate the convergence rate, Zhao and Huang [22] proposed the BiCORSTAB2 method. Under an unified generalized framwork, Zhao and Huang et al. deduced the generalized product-type BiCOR (GPBiCOR) method [23]. Numerical experiments from signal deconvolution show that the GPBiCOR method is effective.

The CORS method is more efficient than the restarted GMRES method on most selected examples, especially coming from realistic RCS calculation [16,21]. However, similarly to the CGS method [6], the CORS method often shows irregular convergence behavior and produces large intermediate residual during the iteration process, which badly affects its convergence rate and accuracy of approximate solutions. Inspired by the generalized CGS (GCGS) method [24], we develop a generalized CORS (GCORS) method which is a new product-type method based on the BiCOR method, where polynomial is products of two nearby BiCOR polynomials. Numerical examples show that this approach may lead to faster convergence as well as to more accurate results. We also show that the CORS and BiCORSTAB methods fit into the framework of the GCORS method.

The remainder of the paper is organized as follows. In Section 2, we give a brief description of the BiCOR method and its properties. The generalized CORS (GCORS) method is derived in Section 3. In Section 4, we present an efficient implementation of the GCORS method, which we will call the generalized CORS2 (GCORS2) method. Finally, numerical experiments are given in Section 5.

Throughout this paper, we use the follow notations. Let the overbar "-" denote the conjugate complex of a scalar, vector or matrix, Z^T and Z^H denote the transpose and the conjugate transpose of a vector or matrix Z, respectively. $\mathbb{P}_m$ denotes the set of complex polynomials $p_m(t)$ of degree m with scalar coefficients satisfying $p_m(0) = 1$. The inner product of two