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STABILITY FOR IMPOSING ABSORBING BOUNDARY CONDITIONS IN THE FINITE ELEMENT SIMULATION OF ACOUSTIC WAVE PROPAGATION*

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Abstract

It is well-known that artificial boundary conditions are crucial for the efficient and accurate computations of wavefields on unbounded domains. In this paper, we investigate stability analysis for the wave equation coupled with the first and the second order absorbing boundary conditions. The computational scheme is also developed. The approach allows the absorbing boundary conditions to be naturally imposed, which makes it easier for us to construct high order schemes for the absorbing boundary conditions. A third-order Lagrange finite element method with mass lumping is applied to obtain the spatial discretization of the wave equation. The resulting scheme is stable and is very efficient since no matrix inversion is needed at each time step. Moreover, we have shown both abstract and explicit conditional stability results for the fully-discrete schemes. The results are helpful for designing computational parameters in computations. Numerical computations are illustrated to show the efficiency and accuracy of our method. In particular, essentially no boundary reflection is seen at the artificial boundaries.

Mathematics subject classification: 35L05, 35L20, 65M06, 65M12, 65M60.
Key words: Stability, Acoustic wave equation, Simulation, Finite element method, Absorbing boundary conditions, Wave operator decomposition.

1. Introduction

Modeling the propagation of seismic waves is a useful step in the interpretation of wave phenomena in complex media. It is also an essential step for inverse problem in seismic exploration. Several kinds of techniques for wave modeling have been developed. These include the finite volume method ([13, 49]), the finite difference method ([2, 20, 32, 43, 52, 53]), the spectral method ([6, 30, 31]), the spectral element method ([28, 29]), the finite element method ([17, 18, 33]) and the discontinuous Galerkin methods ([9, 10, 12, 14, 15]).

The finite difference method is a popular numerical technique because it is relatively easy to implement and has high computational efficiency. The wave modeling with the finite difference

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method in seismology was realized early in 70s ([2]). Since then, various finite difference schemes for wave modeling are proposed. For example, Dablain [20] proposed the high-order difference schemes. Virieux [43] investigated the stagger-grid difference scheme which has some advantages in physical aspects. Sei [36] generalized a family of high order finite difference schemes for the computation of elastic waves. Zhang et al. [52] proposed a new high accuracy locally one-dimensional scheme for the wave equation. The spectral element method was introduced firstly in computational fluid dynamic ([35]). It have been successfully applied to seismic wave simulation ([28, 29]). However, its computational mesh is usually quadrilateral grid in 2D case or hexahedral in 3D case.

The finite element method (FEM) especially the triangular element has a distinctive advantage of being able to handle problems with complex domains. Thus, the FEM has a potential and room for development in seismic wave simulations. However, the high order FEM is still not widely used in the simulation of seismic waves and the main reason is that it requires the inversion of the mass matrix at each extrapolation time step. This implies that the FEM has very low computational efficiency especially when many extrapolation time steps are required. The advantage of FEM is its good adaptability to various velocity models with high complexity. Finite elements with simplices fit better the polygonal shaped domains and sharp contrasts in velocity models. The FEM requires the solution of a large sparse linear system of equations, which makes the method costly. This cost can be avoided by mass lumping ([54]), a technique that replaces the large linear system by a diagonal matrix. For the low order methods such as the linear Lagrange element, the mass lumping can be implemented by using the quadratic rules for numerical integration. But it is not obvious how mass lumping is implemented for high order methods such as the quadratic Lagrange element. As high order accuracy is desired in wave simulation, we will adopt the third-order Lagrange element in our computations which preserves the accuracy and at the same time allows mass lumping ([17, 18]).

In numerical simulation of wave propagation, the imposition of artificial boundary introduces spurious reflections which will devastate the accuracy of numerical solutions. Although the problem can be overcome by increasing the size of the computational domain, it is not always feasible because it increases the amount of computations. In order to eliminate the boundary reflections, absorbing boundary conditions are desirable in wave modeling. There are several kinds of absorbing boundary conditions (ABCs) (see e.g., [5, 7, 14–16, 22, 25, 26, 37]). Smith [37] proposed a nonreflection plane boundary, which is easily implemented for finite difference and finite element calculations. Clayton and Engquist [16] proposed the ABCs based on the paraxial approximations of the acoustic or elastic equations. Another approach is to add damping layer to the boundaries ([7]). The waves entering this damping layer will be absorbed. The perfectly matched layer (PML) method is based on the use of an absorbing layer especially designed to absorb without reflection waves ([5]). These ABCs have been widely used in the finite difference method. Chung (et al., [14, 15]) considered the ABCs in wave simulation with the optimal discontinuous Galerkin methods. In this paper, we focus on stability analysis for implementing the ABCs with the high order Lagrange finite element. The computational scheme is also developed. The ABCs based on the factorization of wave equation are reviewed and the variational framework, which imposes the ABCs weakly, is derived. The spatial discretization of the weak form of wave equation including the embedded boundary conditions has high spatial accuracy. Moreover, we obtain and prove new abstract and explicit stability conditions for the proposed computational scheme.