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EFFICIENT BOX-CONSTRAINED TV-TYPE-l^1 ALGORITHMS FOR RESTORING IMAGES WITH IMPULSE NOISE*

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Abstract

In this paper, we study the restoration of images simultaneously corrupted by blur and impulse noise via variational approach with a box constraint on the pixel values of an image. In the literature, the TV-l^1 variational model which contains a total variation (TV) regularization term and an l^1 data-fidelity term, has been proposed and developed. Several numerical methods have been studied and experimental results have shown that these methods lead to very promising results. However, these numerical methods are designed based on approximation or penalty approaches, and do not consider the box constraint. The addition of the box constraint makes the problem more difficult to handle. The main contribution of this paper is to develop numerical algorithms based on the derivation of exact total variation and the use of proximal operators. Both one-phase and two-phase methods are considered, and both TV and nonlocal TV versions are designed. The box constraint $[0, 1]$ on the pixel values of an image can be efficiently handled by the proposed algorithms. The numerical experiments demonstrate that the proposed methods are efficient in computational time and effective in restoring images with impulse noise.

Mathematics subject classification: 65J22, 65K10, 68U10.
Key words: Image restoration, Impulse noise, Total variation, Nonlocal total variation, Proximal Operators.

1. Introduction

In many real applications, the observed image is the degraded version of the true image. Therefore, image restoration is one of the fundamental tasks in image processing, and it plays an important role in many applications. Indeed, image restoration is a typical inverse problem, and many approaches [8, 22, 40, 42, 47] are proposed to tackle this task.

In this paper, we study the restoration of images corrupted by blur and impulse noise simultaneously via the variational approach with a box constraint on the pixel values of an

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image. Note that usually image blur comes from many facts such as camera shaking, object movement or an out of focus lens. Moreover, impulse noise is often found and modeled in digital storage and transmission. Since each image pixel stands for light intensity whose value is nonnegative and finite, one would like to recover the image with pixel values in the same range. In this paper, we assume that the values of the all pixels of the images are in the range $[0,1]$ for simplicity. The importance of box constraints was emphasized by [26, 31, 35, 41] and references therein.

Assume that both the true image u and the corrupted image g are defined on $\Omega \subseteq \mathbb{R}^{m\times n}$, the image degradation model can be written as follows:

$$g = N_{imp}(Au), \tag{1.1}$$

where A is a known linear operator from $\mathbb{R}^{m\times n}$ to $\mathbb{R}^{m\times n}$, N_{imp} represents the degradation by the impulse noise.

In the literature, there are two common types of impulse noise: salt-and-pepper noise and random-valued noise [10, 11, 46]. Suppose that the noise level is r $(0 \le r \le 1)$, the model of corruption by the salt-and-pepper noise at location (i,j) can be defined as:

$$g_{ij} = \begin{cases} 0, & \text{with probability } r/2, \\ 1, & \text{with probability } r/2, \\ (Au)_{ij}, & \text{with probability } 1-r, \end{cases}$$

and the model of corruption by the random-valued noise at location (i,j) is:

$$g_{ij} = \begin{cases} d_{ij}, & \text{with probability } r, \\ (Au)_{ij}, & \text{with probability } 1-r, \end{cases} \tag{1.2}$$

where the values $d_{ij} \in [0,1]$ come from an independent and identically distributed uniform random numbers in $[0,1]$. One can obviously observe that both salt-and-pepper noise and the random-valued noise corrupt a certain number of image pixels and keeps the remaining pixels uncorrupted. As the value $d_{i,j}$ in (1.2) poses another uncertainty, one can understand that the random-valued noise is more difficult to handle than the salt-and-pepper noise (see [46]).

In the past, various approaches have been proposed for removing the impulse noise. One is the filtering technique, and the most popular filter is the median filter [39] which is efficient and easy to be implemented. However, the median filter often makes the recovered image blurry because it accomplishes the filtering task via replacing each pixel in the image by the median value in a window centered on it. Thus, some modified versions of the median filter are proposed, such as the weighted median filter [7], the adaptive median filter [28], the multi-state median filter [12], the center-weighted median filter [32], etc. Although these modified filters can preserve more details in the recovered image than the median filter, they still cannot preserve the image edges well especially when the noise level is high.

Recently, by combing an l^1 data-fidelity term and an edge-preserving regularization term, Nikolova [38] proposed a method to better preserve the edge information based on the variational approach. It is shown that the l^1 data-fidelity term has rather good performance on detecting outlier and removing impulse noise [37, 38]. However, this method changes the values of the uncorrupted pixels. Based on the self-similarity of natural images [16], Xiao et al. [46] proposed a powerful patch-based method to remove mixed Gaussian-impulse noise. They obtain much better results than the compared methods.