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AN ACCELERATED WAVEFORM RELAXATION APPROACH BASED ON MODEL ORDER REDUCTION FOR LARGE COUPLING SYSTEMS*

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Abstract

In this paper, we present an accelerated simulation approach on waveform relaxation using Krylov subspace for a large time-dependent system composed of some subsystems. This approach first allows these subsystems to be decoupled by waveform relaxation. Then the Arnoldi procedure based on Krylov subspace is provided to accelerate the simulation of the decoupled subsystems independently. For the new approach, the convergent conditions on waveform relaxation are derived. The robust behavior is also successfully illustrated via numerical examples.

Mathematics subject classification: 78M34, 65Pxx.
Key words: Large coupling systems, Waveform relaxation, Model order reduction, Krylov subspace, Convergence analysis, Accelerating technique.

1. Introduction

Time-dependent systems are widely used to model and simulate complex physical processes. With the rapid development of the very large-scale integration technology, the dimension of time-dependent systems often becomes very large. For simulating such systems, it becomes extremely important to seek robust numerical simulation methods. Due to this fact, how to numerically solve large time-dependent systems has attracted extensive attention.

In this paper, we consider a time-dependent system composed of k subsystems. For convenience sake, we assume that each subsystem is a linear time-invariant system described as

$$E_j\frac{dx_j(t)}{dt} = A_jx_j(t) + B_ju_j(t),\ y_j(t) = C_jx_j(t),\ j = 1, 2, \cdots, k, \tag{1.1}$$

with the initial conditions $x_j(t_0)$, and

$$\begin{cases} u_j(t) = F_{j1}y_1(t) + F_{j2}y_2(t) + \cdots + F_{jk}y_k(t) + G_ju(t), \\ y(t) = H_1y_1(t) + H_2y_2(t) + \cdots + H_ky_k(t), \end{cases} \tag{1.2}$$

where $E_j, A_j \in \mathbb{R}^{n_j\times n_j}(j = 1, 2, \cdots, k)$, $B_j \in \mathbb{R}^{n_j\times m_j}$, $C_j \in \mathbb{R}^{p_j\times n_j}$, $F_{ji} \in \mathbb{R}^{m_j\times p_i}(i = 1, 2, \cdots, k)$, $G_j \in \mathbb{R}^{m_j\times m}$, $H_j \in \mathbb{R}^{p\times p_j}$, $x_j(t) \in \mathbb{R}^{n_j}$ are internal state variables, $u_j(t) \in \mathbb{R}^{m_j}$ are internal inputs, $y_j(t) \in \mathbb{R}^{p_j}$ are internal outputs, $u(t) \in \mathbb{R}^m$ is an external input, and $y(t) \in \mathbb{R}^p$ is an external output. To our knowledge, this kind of system frequently arises in

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numerous research areas such as circuit simulation, control models, and discretizations of partial differential equations.

In the literature, waveform relaxation (WR), also known as dynamic iteration, is an effective technique to solve coupled systems described by ordinary differential equations and partial differential equations, for details see [4–6, 9, 10]. The WR technique allows coupled systems to be independently solved with its own time step length. Two typical WR schemes are the Jacobi and Gauss-Seidel relaxation processes. For simulating the system (1.1), the WR technique is an effective decoupling method. For example, for the j-th ($j = 1, 2, \cdots, k$) subsystem of (1.1), an iterative form of WR can be constructed as

$$\begin{aligned} &E_{1j}\frac{dx_j^{(l+1)}(t)}{dt} - \Big(A_{1j} + B_{1j}F_{1,jj}C_{1j}\Big)x_j^{(l+1)}(t) \\ =&E_{2j}\frac{dx_j^{(l)}(t)}{dt} - \Big(A_{2j} + B_{2j}F_{2,jj}C_{2j}\Big)x_j^{(l)}(t) + B_j\Big(\sum_{i=1,i\neq j}^{k} F_{ji}C_i x_i^{(l)}(t) + G_j u(t)\Big), \end{aligned} \tag{1.3}$$

where $E_{1j} - E_{2j} = E_j$, $(A_{1j} + B_{1j}F_{1,jj}C_{1j}) - (A_{2j} + B_{2j}F_{2,jj}C_{2j}) = A_j + B_jF_{jj}C_j$, l is a nonnegative integer, $x_j^{(l+1)}(t_0)(j = 1, 2, \cdots, k)$ are initial conditions, and the functions $x_j^{(0)}(\cdot)$ are initial guesses. Numerical algorithms with WR suit well for parallel processing. In addition, model order reduction is another effective technique which seeks to replace a very large-scale integration system by a system of substantially lower order. There are two main kinds of model order reduction methods. The first one is the Krylov subspace method, for details see [7, 12]. The other one is the balanced truncation reduction method, e.g., see [7, 13, 16]. For nonlinear systems, some model order reduction methods are discussed in [1, 2, 15]. Some work on model order reduction can also be referred to [11, 14].

Instead of direct numerical simulation of the system (1.1), we use the Krylov subspace model order reduction technique to construct a reduced system as follows

$$\tilde{E}_j\frac{d\tilde{x}_j(t)}{dt} = \tilde{A}_j\tilde{x}_j(t) + \tilde{B}_j\tilde{u}_j(t),\ \tilde{y}_j(t) = \tilde{C}_j\tilde{x}_j(t),\ j = 1, 2, \cdots, k, \tag{1.4}$$

where $\tilde{x}_j(t) \in \mathbb{R}^{q_j}$, $\tilde{u}_j(t) \in \mathbb{R}^{m_j}$, $\tilde{y}_j(t) \in \mathbb{R}^{p_j}$, $\tilde{E}_j, \tilde{A}_j \in \mathbb{R}^{q_j\times q_j}$, $\tilde{B}_j \in \mathbb{R}^{q_j\times m_j}$, $\tilde{C}_j \in \mathbb{R}^{p_j\times q_j}$, $q_j \ll n_j$, and

$$\tilde{u}_j(t) = \sum_{i=1,i\neq j}^{k} F_{ji}\tilde{y}_i(t) + G_j u(t), \quad \tilde{y}(t) = \sum_{i=1}^{k} H_i\tilde{y}_i(t).$$

Our method combining WR with Krylov subspace seeks to remedy the shortcomings of WR, such as the poor convergence property and expensive computational costs. The WR technique not only gives a decoupling method for the system (1.1) but also offers a new model order reduction strategy based on Krylov subspace. Some concrete Krylov subspaces which can bring the original system (1.1) to the reduced system (1.4) may be constructed based on the iterative process (1.3).

The outline of this paper is organized as follows. In Section 2, we present some basic properties of solving the system (1.1), and discuss the decoupling of this system by the WR technique. Moreover, for the system of index one, the convergence condition of the WR solutions is derived. In Section 3, we reduce each independent subsystem to a system with lower order and analyze the convergence of the WR solutions for the reduced system of index one. The moment matching property is also analyzed in Section 3. In Section 4, we present a structure-preserving algorithm which preserves the differential-algebraic structure of the original system.