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A SIMPLE PRECONDITIONED DOMAIN DECOMPOSITION METHOD FOR ELECTROMAGNETIC SCATTERING PROBLEMS*

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Abstract

We present a domain decomposition method (DDM) devoted to the iterative solution of time-harmonic electromagnetic scattering problems, involving large and resonant cavities. This DDM uses the electric field integral equation (EFIE) for the solution of Maxwell problems in both interior and exterior subdomains, and we propose a simple preconditioner for the global method, based on the single layer operator restricted to the fictitious interface between the two subdomains.

Mathematics subject classification: 65F08, 65N38, 65R20.
Key words: Electromagnetism, integral equations methods, Domain decomposition methods, Preconditioning, Cavities.

1. Introduction

Solving scattering Maxwell problems in harmonic regime can be achieved with various methods, among which integral equations (which lead to the so-called boundary element methods) have proven their efficiency. Their main advantage is that they allow to replace a problem posed on the whole space by an equation posed on the surface of the scattering obstacle, reducing a three-dimensional problem to a bi-dimensional one. With the development of such methods, several difficulties arose successively:

- These formulations lead classically to linear systems involving dense matrices (in contrast with finite element methods, for instance). Several methods among which the most famous is probably the FMM (Fast Multipole Method) [28], [29] have been used to circumvent this difficulty.

- There might exist irregular frequencies for which the problem is ill posed [26]. This is typically the case for the so-called EFIE and MFIE formulations. Other types of formulations (e.g., the CFIE) are instead well-posed at any frequency [7].

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- The desire to deal with high frequency problems imposes to use fine discretizations of the equations and consequently to solve large linear systems. This prevents the use of direct solvers, and one usually employs iterative methods. On the one hand, this needs a fast matrix-vector multiplication (which is often realized through the FMM), while on the other hand iterative methods become sensible to the condition number of the system. It has been shown that the underlying systems arising from integral equations are usually badly conditioned and there is a need to develop preconditioning strategies in order to accelerate the convergence of the iterative solver [10,22,30,31]. For instance, the so-called GCSIE methodology has been developed which turns out to be particularly efficient in the case where the object has no cavities and no singularities, by building intrinsically well conditioned integral equations [1,6,14,23,27].

Nevertheless, when facing realistic problems, one has to treat large objects with complex geometries and new problems are encountered. In this paper, we address the important issue of resonant cavities, motivating the use of a domain decomposition method. Indeed, this is a particularly crucial problem in stealth applications as one needs to take into account the existence of large and resonant cavities, such as air intakes, or cockpits for aircrafts. In classical numerical computations of radar cross sections, these cavities are usually closed in order to avoid the poor convergence of the algorithms [1] giving unrealistic results.

In this paper, we explore a new strategy to deal with this problem. Indeed, we intend to use a domain decomposition method (DDM) in order to split the exterior domain into two subdomains one of which being the cavity. The aim is to decouple the exterior problem (without any cavity) from the problem with boundaries (the cavity itself). This introduces an artificial interface Σ between these subdomains and a new coupling problem posed on Σ (see, Fig. 1.1). For simplicity, we here use on each subdomain the EFIE to solve the corresponding subproblems and to couple the solutions on Σ. This naive DDM algorithm turns out to converge badly. In a latter part we propose a preconditioning technique to accelerate significantly the solution of the DDM.

Historically, the first domain decomposition methods for Helmholtz or Maxwell problems were applied using a finite element method (FEM) in the interior bounded subdomains and a boundary element method (BEM) in the exterior unbounded domain. For instance, Hiptmair considers FEM-BEM methods, first applied to acoustic problems [20] and then to electromagnetic problems [21]. For Helmholtz transmission problems, domain decomposition methods have been used by Balin, Bendali and Collino [4] to specifically treat the case of an electrically deep cavity, and an integral preconditioner using the Calderón formulas has been developed by Antoine and Boubendir [3]. For Maxwell transmission problems, Balin, Bendali and Millot [5] on the one hand, Collino and Millot on the other hand [11,12] propose algebraic preconditioners which use overlapping or nonoverlapping domain decomposition techniques. In iterative domain decomposition techniques, which are split into overlapping and nonoverlapping DDM, the subdomains classically exchange Dirichlet or Neumann data. A substantial improvement using absorbing boundary conditions is made by Després [15,16]. The Schwarz method, originally used with Dirichlet or Neumann conditions for overlapping domains, is then adapted by Gander, Halpern and Nataf [19], to nonoverlapping subdomains with more general conditions, of Robin type. The resulting algorithm converges with a high convergence rate for the wave equation in dimension 1. Gander, Halpern and Magoulès [18] optimize the method by taking more general conditions, for the Helmholtz problem in dimension 2. Eventually, Dolean, Gander and Gerardo-Giorda [17] adapt it to obtain a Schwarz optimized method for the harmonic