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INVERSION OF ELECTRON TOMOGRAPHY IMAGES USING L^2-GRADIENT FLOWS —- COMPUTATIONAL METHODS*

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Abstract

In this paper, we present a stable, reliable and robust method for reconstructing a three dimensional density function from a set of two dimensional electric tomographic images. By minimizing an energy functional consisting of a fidelity term and a regularization term, an L^2-gradient flow is derived. The flow is integrated by a finite element method in the spatial direction and an explicit Euler scheme in temporal direction. The experimental results show that the proposed method is efficient and effective.

Mathematics subject classification: 65D17.
Key words: Computational Inversion, Reconstruction, Electric Tomography.

1. Introduction

Electron microscopy (EM) is a preferred tool for structural biologists to visualize three-dimensional structures of molecular and cellular complexes in-vitro. Electron tomography (ET) involves acquiring planar EM images of the biological sample from different projection angles and reconstructing a 3D image from these projections [9]. The forward operation assumes the Born approximation of electron beam-specimen interaction [8]. This holds true only when the specimen is weakly scattering. The scattering function depends on the electrostatic properties of the specimen. Images also suffer from effects of blurring owing to specimen deformation during exposure to the electron beam. Another challenge is that the angle of rotation cannot exceed $\pm 70^\circ$(from the horizontal plane) since the beam length through the specimen becomes large at higher tilt angles as it passes through the plane of the sample resulting in poor images. Hence projections are available only for a limited range of tilt angles, which is known as the missing wedge problem [14]. Since there is no unique solution to the inverse reconstruction problem it makes image processing and visualization of ET data a huge challenge. Typically, acquisition and reconstruction processes contribute largely to the error in modeling biological specimens from EM images.

In this paper, we propose an iterative variational reconstruction method. By minimizing an energy functional consisting of a fidelity term and a regularization term, an L^2-gradient flow is derived. The flow is integrated by a finite element method in the spatial direction and

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an explicit Euler scheme in the temporal direction. The experimental results show that the proposed method is stable, reliable and robust. By stable, we mean the reconstruction result does not depend on the initialization of the iterative scheme, and the reconstruction result is unique. By reliable, we mean the the projected images of the reconstruction result are as close to the given images as possible. The robustness implies that the reconstruction is not sensitive to small noise in the 2D images.

Our results are demonstrated on a phantom data as well as tomographic images of the AIDS virus complex, specifically the Simian Immunodeficiency Virus (SIV) [4] which is similar to the HIV. Liu, et al. [15] describe the structure of the HIV virus and the action of the spike protein in infecting the host. The phantom data is a spherical shell with nonzero thickness.

The rest of this paper is organized as follows. In Section 2, we review existing 3D EM reconstruction techniques. In Section 3, we establish notation and define several parameters and data structures including image size, B-spline basis function grids and volume grids are described. Sections 4 and 5 provides details of our algorithms. Section 6 gives some illustrative examples. Section 7 contains the conclusion of this paper. Additionally important mathematical facts and proofs on the existence and uniqueness of the reconstruction are given in the Appendix.

2. A Review of Existing ET Reconstruction Techniques

Reconstruction methods try to invert the following system of equations that describe the projection process, also called the *forward problem*,

$$I_i = \sum_{n=1}^{N} w_{in} f_n, \quad i = 0, 1, \cdots,$$

where I_i is the measured projection data of the i-th projection ray through a space volume with N voxels. w_{in} is a weight that describes the contribution of the n-th voxel to the i-th projection. Given I_i and w_{in}, the goal of the *inverse problem* is to compute f_n.

2.1. Algebraic Reconstruction Technique (ART)

An iterative approach to solve the inverse problem is to use the Kaczmarz method [12,13,16]. In this technique each tomogram image is treated like a hyperplane with the equation $\mathbf{w}_i{}^T\mathbf{f} = I_i$ in an N dimensional space, where

$$\mathbf{w}_i = [w_{i,1}, \cdots, w_{i,N}]^T, \quad \mathbf{f} = [f_1, \cdots, f_N]^T.$$

A solution to the entire equation system for all tomograms $I = [I_0, I_1 \cdots]^T$ is a point in the N dimensional space where all the hyperplanes intersect. To solve for this solution using an iterative method, the algorithm starts off with a random solution (typically zero) $\mathbf{f}^{(0)}$. This initial solution is then projected onto the first hyperplane. The projected point on the first hyperplane is then projected onto the next hyperplane and so on for all the image values I_i. The iterative process will converge to the point of intersection of all the hyperplanes [13]. If there is no unique solution which is typically the case, then the iterative scheme will not converge and will oscillate. To work around this problem a regularizing term λ is used in the update equation [5]:

$$\mathbf{f}^{(i+1)} = \mathbf{f}^{(i)} - \lambda \frac{\mathbf{w}_{i+1}^T \mathbf{f}^{(i)} - I_{i+1}}{||\mathbf{w}_{i+1}||^2} \mathbf{w}_{i+1},$$

where $\mathbf{f}^{(i)} = [f_1^{(i)}, \cdots, f_N^{(i)}]^T$.