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A FAST SIMPLEX ALGORITHM FOR LINEAR PROGRAMMING*

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Abstract

Recently, computational results demonstrated remarkable superiority of a so-called "largest-distance" rule and "nested pricing" rule to other major rules commonly used in practice, such as Dantzig's original rule, the steepest-edge rule and Devex rule. Our computational experiments show that the simplex algorithm using a combination of these rules turned out to be even more efficient.

Mathematics subject classification: 65K05, 90C05.
Key words: Large-scale linear programming, Simplex algorithm, Pivot rule, Nested, Largest-distance, Scaling.

1. Introduction

Consider the linear programming (LP) problem in the standard form

$$\begin{aligned} &\text{minimize} && c^T x \\ &\text{subject to} && Ax = b, \quad x \geq 0, \end{aligned} \tag{1.1}$$

where $A \in \mathbb{R}^{m \times n}$ $(m < n)$ and $\operatorname{rank}(A) = m$. It will be a simple matter to extend results of this paper to more general LP problems with bounds and ranges.

The pivot rule that is employed to select an index to enter the basis is crucial to computational efficiency of the simplex algorithm, since it essentially determines the number of iterations required for solving LP problems. As a result, a variety of pivot rules have been proposed and tested from time to time (for a survey, see [8] or [13]). Among them, the steepest-edge rule [4, 5] and its approximation, Devex rule [6], are now accepted to be as the best, and are therefore commonly used in commercial packages, such as CPLEX [1, 7].

Recently, Pan reported very encouraging computational results on the largest-distance rule [10] and the nested pricing rule [11, 12] against major commonly used rules, such as Dantzig's original rule as well as the steepest-edge rule and Devex rule. Over 80 test problems, a largest-distance rule yields run times that are reduced by an average factor of 3.24, while a nested pricing rule yields run times reduced by an average factor of 5.73, compared to the Devex rule.

It has been unknown what will happen if the nested pricing rule and the largest-distance rule are put together. For this purpose, we have conducted computational tests on a combination of the two rules with the same test sets, i.e., the 48 largest Netlib problems in terms of the number of rows and columns, all of the 16 Kennington problems, and the 17 largest BPMPD problems in terms of more than 500KB in compressed form. Computational results turned out

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to be even more favorable than a single of them used: it outperformed the Devex rule by an average run time factor as high as 7.27.

In the remaining part of this section, we review briefly the largest-distance rule and the nested pricing rule. In Section 2, we describe the new rule. In Section 3, we report computational results and make final remarks.

Let B be the current basis and N the associated nonbasis. Without confusion, denote *basic* and *nonbasic* index sets again by B and N, respectively. The reduced costs associated with nonbasic indices may be computed by

$$\bar{c}_N = c_N - N^T\pi, \qquad B^T\pi = c_B. \tag{1.2}$$

If index set

$$J = \left\{ j \mid \bar{c}_j < 0,\ j \in N \right\} \tag{1.3}$$

is nonempty, Dantzig's original rule [2, 3] selects an entering index q such that

$$\bar{c}_q = \min \left\{ \bar{c}_j \mid j \in J \right\} < 0. \tag{1.4}$$

1.1. Largest-distance rule

The determination of q is not invariant for scaling. In fact, it is seen from (1.2) and (1.4) that quantities π and $\bar{c}_N$, and hence index q are all dependent of norms of columns of the coefficient matrix A. To eliminates such dependence, the largest-distance rule [10] uses reduced costs normalized by norms of corresponding columns.

1.2. Nested pricing rule

At each iteration, the nested pricing rule [11, 12] gives indices in a subset of N priority to become basic. Pricing is first conducted on it to determine a reduced cost by some criterion. If one is found significantly negative, then the associated index is selected to enter B. If not, the same is done with the remaining set; if no such one is found, optimality is declared.

2. Nested Largest-Distance Rule

To make further progress, we combine the largest-distance rule and the nest pricing rule as follows.

Rule 2.1. Let an optimality tolerance $\epsilon > 0$ be given. Set $J = N$ initially.

1. If

$$\widehat{J} \triangleq \left\{ j \ \mid \ \bar{c}_j/\|a_j\| < -\epsilon,\ j \in J \right\} \tag{2.1}$$

is nonempty, go to step 4; else,

2. If

$$\widehat{J} \triangleq \left\{ j \ \mid \ \bar{c}_j/\|a_j\| < -\epsilon,\ j \in N \backslash J \right\} \tag{2.2}$$

is nonempty, go to step 4; else