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A NEW TRUST-REGION ALGORITHM FOR NONLINEAR CONSTRAINED OPTIMIZATION*

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Abstract

We propose a new trust region algorithm for nonlinear constrained optimization problems. In each iteration of our algorithm, the trial step is computed by minimizing a quadratic approximation to the augmented Lagrange function in the trust region. The augmented Lagrange function is also used as a merit function to decide whether the trial step should be accepted. Our method extends the traditional trust region approach by combining a filter technique into the rules for accepting trial steps so that a trial step could still be accepted even when it is rejected by the traditional rule based on merit function reduction. An estimate of the Lagrange multiplier is updated at each iteration, and the penalty parameter is updated to force sufficient reduction in the norm of the constraint violations. Active set technique is used to handle the inequality constraints. Numerical results for a set of constrained problems from the CUTEr collection are also reported.

Mathematics subject classification: 90C30, 65K05.
Key words: Trust region method, Augmented Lagrange function, Filter method, active set.

1. Introduction

This paper presents a new trust region algorithm for general constrained optimization problems having the form:

$$\min \quad f(x) \tag{1.1a}$$

$$\text{subject to} \quad c_i(x) = 0, \qquad i \in \mathcal{E} = \{1, ..., m_e\} \tag{1.1b}$$

$$c_i(x) \geq 0, \qquad i \in \mathcal{I} = \{m_e + 1, ..., m\} \tag{1.1c}$$

where $f : \Re^n \to \Re$ and $\mathbf{c} : \Re^{\mathbf{n}} \to \Re^{\mathbf{m}}$ are twice continuously differentiable functions.

The trust-region methods are a class of numerical methods for optimization. While line search type methods search the next iteration in a line, trust region methods try to find the next iteration point within a region. Such a region is called the trust region and it is normally a set (say, a ball or box) centered at the current iterate. The essential parts of a trust region method are finding the trial step in the trust region and deciding whether the trial step should be accepted [28].

The trial step of a trust region method is normally computed by solving a trust region subproblem. There are mainly three different approaches. The null space type method decomposes the trial step into a range space step and a null space step with the range space step

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reducing the constraint violations and the null space step decreasing the Lagrange function in the null space (for example, see [1, 18, 26]). The second type of trust region subproblems is the the two-ball subproblem, which minimizes a quadratic approximation to the objective function subject to a reduction of the norm of the linearized constraints (see, e.g., [2, 21]). The third kind of trust region subproblems can be derived by exact penalty functions, which minimize an approximation to some penalty function. Such subproblems include the SL_1QP subproblem [10] and L_∞ subproblem [27]. The subproblem we use in our new method belongs to the third type. Our subproblem is to minimize an approximate augmented Lagrangian function. Augmented Lagrangian function [19] is an exact penalty function if the Lagrange multiplier is exact and the penalty parameter is sufficiently large.

Recently, merit-function-free algorithms have been attracted much attention from researchers, for example see [12, 25]. The basic idea of such algorithms is to regard the constrained optimization problem as a two objective problem. One is to decrease the original objective function while the other is to decrease the constraint violations. Algorithms based on merit functions normally require a monotone decrease in a merit function, while merit-function-free algorithms use non-monotone decrease conditions on the original objective function and the constraint violations. In our algorithm, we also use the filter idea and a trial step will not be thrown away unless it not only does not reduce the merit function, but also not accepted by a filter.

This paper is organized as follows. The next section introduces the motivation and the basic idea of our new algorithm based on the equality constrained problem. In section 3 we extend our algorithm to general constrained problems by employing the active set technique. Section 4 presents some numerical results for problems from the CUTEr collection [14] and gives a brief conclusion.

Throughout the paper $\|\cdot\|$ denotes the Euclidean norm. We denote the gradient of f by $g = g(x) = \nabla f(x)$, the Jacobian of the constraints by $A = A(x) = (\nabla c_1(x), \nabla c_2(x), \cdots, \nabla c_m(x))$. Superscripts $^{(k)}$ refer to iteration indices and $f^{(k)}$ is taken to mean $f(x^{(k)})$ etc.Subscripts $_k$ refer to elements of vector. Quantities related to a local solution are superscripted by $*$.

2. An Algorithm for Equality Constrained Optimization

In this section, we give a new algorithm for the equality constrained optimization

$$\min \quad f(x) \tag{2.1a}$$

$$\text{subject to} \quad c_i(x) = 0, \qquad i \in \mathcal{E}. \tag{2.1b}$$

Before giving the detailed descriptions of our algorithm, we need to address some issues such as the calculations of the trial step, techniques for updating the Lagrange multiplier and the penalty parameter, and the criteria for accepting the trial step.

2.1. Computing the trust region step

The trust region step is computed by minimizing a quadratic function which approximates the augmented Lagrangian function. The augmented Lagrangian function has the form

$$\Phi(x, \lambda, \sigma) = f(x) - \lambda^T c(x) + \sigma \|c(x)\|^2, \tag{2.2}$$

where $\lambda \in \Re^{|\mathcal{E}|}$ is the Lagrange multiplier and $\sigma \geq 0$ is the penalty parameter.