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ADAPTIVITY IN SPACE AND TIME FOR MAGNETOQUASISTATICS*

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Abstract

This paper addresses fully space-time adaptive magnetic field computations. We describe an adaptive Whitney finite element method for solving the magnetoquasistatic formulation of Maxwell's equations on unstructured 3D tetrahedral grids. Spatial mesh refinement and coarsening are based on hierarchical error estimators especially designed for combining tetrahedral $\boldsymbol{H}(\mathbf{curl})$-conforming edge elements in space with linearly implicit Rosenbrock methods in time. An embedding technique is applied to get efficiency in time through variable time steps. Finally, we present numerical results for the magnetic recording write head benchmark problem proposed by the Storage Research Consortium in Japan.

Mathematics subject classification: 65M60, 65L06, 78M10
Key words: Magnetoquasistatics, Space-time adaptivity, Edge elements, Rosenbrock methods, Hierarchical error estimator, SRC benchmark problem.

1. Introduction

The magnetoquasistatic approximation (MQS) arises from Maxwell's equations by dropping the displacement current. This is reasonable for many electrical machines, generators and transformers which work in the low-frequency high-conductivity range. Wave propagation can then be neglected and vanishing tangential traces are used for artificial boundary conditions [18].

In this work we develop a fully adaptive algorithm to solve general three-dimensional nonlinear MQS problems. The local accuracy of the numerical solution is controlled by means of a posteriori error estimates in space and time. In the past, computational electromagnetics has mainly focused on efficiency by (i) applying advanced multigrid algorithm with optimal complexity to solve large scale linear systems, e.g., [11, 17, 19, 28, 34], (ii) adapting spatial grids by means of a posteriori error estimators [8, 12, 32, 37], and to some extent by (iii) optimizing time grids in accordance with local error control [13, 15, 16, 40]. An interesting alternative approach is the goal-oriented weighted dual method [10]. Often, in addition, highly parallelized strategies

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are applied. On the other side the reliability question, that is, how accurate is the numerical solution computed, has received much less attention in MQS simulation.

There is nowadays an increasing emphasis on all aspects of adaptively generating a space-time grid that evolves with the solution. Equally important is the development of efficient higher-order one-step integration methods which can handle very stiff differential-algebraic electromagnetic problems and which allow us to accommodate a grid in each time step without any specific difficulties. Combined space-time adaptivity is widely used in computational fluid dynamics and thermodynamics, see, e.g., [39].

Recently, first investigations for space and time adaptive MQS solvers have been made in [42] where first-order approximations in time and space are considered and in [41] where higher-order embedded SDIRK-methods are used for first-order spatial discretizations. In [31] a new variable step-size one-step Rosenbrock methods ROS3PL is coupled with lowest-order edge elements to solve linear MQS problems. Here, we extent the latter approach to nonlinear material laws. We wish to adaptively refine space-time grids in order to capture local effects efficiently and reliably in accordance to imposed temporal and spatial tolerances. We apply the adaptive Rothe method based on the discretization sequence first in time then in space, in contrast to the usual Method of Lines approach (see, e.g., [29] and references therein). The spatial discretization is considered as a perturbation of the time integration process. Implementations have been done in the KARDOS library [2, 23], which provides a suitable programming environment for adaptive algorithms to solve nonlinear time-dependent PDEs.

2. Problem Class

Introducing a vector potential $\boldsymbol{A}(\boldsymbol{x},t)$ for the magnetic induction $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$, we consider the equations of magnetoquasistatics for isotropic materials in the form

$$\begin{aligned} \sigma\partial_t\boldsymbol{A} + \boldsymbol{\nabla}\times\left(\mu^{-1}(|\,\boldsymbol{\nabla}\times\boldsymbol{A}|)\,\boldsymbol{\nabla}\times\boldsymbol{A}\right) &= \boldsymbol{J}_s, \text{ in } \Omega\times(0,T], \\ \boldsymbol{A}\times\boldsymbol{n} &= \boldsymbol{0}, \text{ on } \partial\Omega\times(0,T], \\ \boldsymbol{A}(\cdot,0) &= \boldsymbol{A}_0, \text{ on } \Omega \end{aligned} \tag{2.1}$$

where σ is the scalar electric conductivity and $\boldsymbol{J}_s(\boldsymbol{x},t)$ denotes the applied current density which has to satisfy the consistency condition $\nabla\cdot\boldsymbol{J}_s = 0$. The scalar magnetic permeability μ is in general nonlinear and is defined by the material relation $\boldsymbol{H} = \mu^{-1}(|\boldsymbol{B}|)\boldsymbol{B}$ between the magnetic field $\boldsymbol{H}$ and the magnetic induction $\boldsymbol{B}$. Here, $|\cdot|$ stands for the usual Euclidean vector norm. Due to physical arguments, the continuous function $\mu^{-1}(s):\mathbb{R}_0^+\to\mathbb{R}^+$ satisfies the following properties [35]:

$$\begin{aligned} &0 < \underline{\mu}^{-1} \le \mu^{-1}(s) \le \mu_0^{-1} \ \text{ for all } s\,, \\ &f(s) = s\mu^{-1}(s) \ \text{ is strictly monotone and Lipschitz continuous,} \end{aligned} \tag{2.2}$$

where $\mu_0 = 4\pi\times10^{-7}Hm^{-1}$ is the permeability in vacuum.

Since there may be insulating regions with $\sigma = 0$, system (2.1) is in general an elliptic-parabolic initial-boundary value problem. The physically relevant quantities which can be derived from $\boldsymbol{A}$ are the magnetic induction $\boldsymbol{B} = \boldsymbol{\nabla}\times\boldsymbol{A}$ and the eddy current density $\boldsymbol{J}_E = -\sigma\partial_t\boldsymbol{A}$. The vector potential formulation (2.1) is widely used in electromagnetic computations since it has no problems with multiple connected conductive domains. However, there are two essential difficulties: the uniqueness of $\boldsymbol{A}$ in parts of the domain where $\sigma = 0$, and the