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AN EFFICIENT MOVING MESH METHOD FOR A MODEL OF TURBULENT FLOW IN CIRCULAR TUBES*

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Abstract

This paper presents an efficient moving mesh method to solve a nonlinear singular problem with an optimal control constrained condition. The physical problem is governed by a new model of turbulent flow in circular tubes proposed by Luo *et al.* using Prandtl's mixing-length theory. Our algorithm is formed by an outer iterative algorithm for handling the optimal control condition and an inner adaptive mesh redistribution algorithm for solving the singular governing equations. We discretize the nonlinear problem by using a upwinding approach, and the resulting nonlinear equations are solved by using the Newton-Raphson method. The mesh is generated and the grid points are moved by using the arc-length equidistribution principle. The numerical results demonstrate that proposed algorithm is effective in capturing the boundary layers associated with the turbulent model.

Mathematics subject classification: 65L10, 65L12.
Key words: Eddy viscosity, Turbulent pipe flow, Boundary layer, Optimal control, Moving mesh.

1. Introduction

The modeling of turbulent flows still plays an important role in computational fluid dynamics because direct simulation of flows are restricted to very simple geometries and low Reynolds number [6, 9, 18, 19]. Development of turbulence model is therefore still an important task and even some semi-empirical means such as the eddy viscosity or Prandtl's mixing length are very helpful to deal with many problems in engineering practice due to their simplicity. Luo *et al.* [15] established a new model of turbulent flow in circular tubes which is an application and improvement of Prandtl's mixing-length theory. The model expresses the single phase flow in circular tubes, which is an optimal parameter control problem governed by a nonlinear singular equation. The model yields many complex mathematical characters such as strong boundary layer. The computational results resulting from the new model are found in good agreement with the experimental results on fluid velocity distribution, eddy viscosity distribution and friction factor. On the mathematical side, the governing equations associate with this model

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are quite complicated, and an effective numerical scheme for finding numerical approximations seems useful.

The difficulties of this problem include the existence of the boundary layers and an optional control condition enforced on the governing equations. To resolve the layers, numerical simulations require extremely fine meshes on the small localized portions of the physical domain. It will become very expensive if a uniform fine mesh is employed. Recent study has demonstrated that moving mesh methods are powerful in resolving large solution variations by increasing the solution accuracy and decreasing the cost of computations, see, e.g., Adaptive moving mesh methods have important applications in solving partial differential equations (PDEs). Up to now, there have been important progresses [1,3,20]. Harten and Hyman [11] began the earliest study of the adaptive methods to improve resolution of discontinuous solutions of hyperbolic equations. After their work, many other moving mesh methods on this direction have been proposed based on combining the variational grid methods with high resolution shock capturing methods, including the so-called moving mesh PDE (MMPDE) approach of moving mesh methods of W. Huang [10], moving finite element methods of Miller [17], and moving finite volume methods [22]. Recently, there have been works on moving mesh methods based on Harmonic maps [8,16,22]. Theoretical results on adaptive mesh arising from equidistribution of a monitor function can be found in [3,4,12–14,21]. In particular, Kopteva [13] derived certain maximum norm a posterior error estimates for one-dimensional singularly perturbed convection-diffusion problems, see also a recent paper [14] for a similar posterior error estimate.

The aim of this paper is to present an efficient and fast numerical method for the turbulent model. The proposed numerical algorithm includes two parts: (i) the outer iterative algorithm is used to solve the optimal control condition and (ii) the inner adaptive mesh redistribution algorithm is used to solve the singular problem. We discretize this nonlinear problem by using upwinding scheme. The discretized nonlinear equations is solved by Newton-Raphson method. The arc-length equidistribution principle is used in the part (ii) above. The numerical examples will be provided to demonstrate the effectiveness of the proposed algorithm.

This paper is organized as follows. In Section 2, we briefly review the model of turbulent flow in circular tubes by employing Prandtl's mixing-length theory. In Section 3, we will present the discrete schemes and algorithms. Numerical experiments will be carried out in Section 4. Some concluding remarks will be presented in the final section.

2. A New Model of Turbulent Flow in Circular Tubes

In this section, we briefly review the background of the model of turbulent flow in circular tubes which was proposed by Luo *et al.* [15]. Moreover, using dimensionless analysis we will derive a complete mathematical description for this model.

Note that the shearing stress of Newtonian fluid for turbulent flow can be described by eddy viscosity with dimensionless analysis [9]. We then have following expression:

$$\frac{d\tilde{u}}{d\phi} = \frac{-\hat{R}\phi}{1+\mu_t/\mu_L}, \tag{2.1}$$

where $\tilde{u}$ is dimensionless time-smoothed velocity, μ_t is eddy viscosity, μ_L is kinematic viscosity, $\hat{R}$ is dimensionless radius of a circular pipe and $\hat{R} = \rho\hat{u}R/\mu$ with R is the tube radius, ρ is the liquid density, $\hat{u}$ is friction velocity, μ is the molecule viscosity, ϕ is dimensionless radial position in a circular pipe, $\phi = 0$ is the center of the tube and $\phi = 1$ corresponds to the wall of