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HANGING NODES IN THE UNIFYING THEORY OF A POSTERIORI FINITE ELEMENT ERROR CONTROL*

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Abstract

A unified a posteriori error analysis has been developed in [18, 21–23] to analyze the finite element error a posteriori under a universal roof. This paper contributes to the finite element meshes with hanging nodes which are required for local mesh-refining. The two-dimensional 1−irregular triangulations into triangles and parallelograms and their combinations are considered with conforming and nonconforming finite element methods named after or by Courant, Q_1, Crouzeix-Raviart, Han, Rannacher-Turek, and others for the Poisson, Stokes and Navier-Lamé equations. The paper provides a unified a priori and a posteriori error analysis for triangulations with hanging nodes of degree ≤ 1 which are fundamental for local mesh refinement in self-adaptive finite element discretisations.

Mathematics subject classification: 65N10, 65N15, 35J25.
Key words: A posteriori, A priori, Finite element, Hanging node, Adaptive algorithm.

1. Introduction

More and more accurate scientific simulations in less and less CPU time on smaller and smaller computational resources are one important new feature in natural sciences, engineering, medicine, and business with huge impact on our modern technological societies. The presently most important area of worldwide scientific activities in the design of more effective and accurate numerical predictions in the computational sciences is the proper mesh-design within the discretisation of partial differential or integral equations.

Nonconforming finite element methods on parallelograms are of particular attraction in computational fluid and solid mechanics because of their conservation properties. Their application in adaptive local mesh-refining algorithms, however, involves a partition with triangles or with hanging nodes of at least first order. This paper is devoted to a universal a priori and a posteriori error analysis for those 1-irregular meshes specified in Section 2 below. Tables 1.1 and 1.2 display practical solutions for the Laplace, Stokes and Navier Lamé equations problem discussed in this paper.

At first glance, the concept of hanging nodes appears straightforward: If a vertex x of a finite element domain (with polygonal boundary) belongs to the interior of an edge E (of another element domain) in the sense that it is a nontrivial convex combination of the end points of E, then z is called a hanging node. However, in case of continuous discrete functions, the

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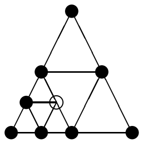
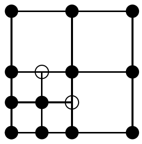
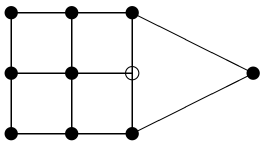
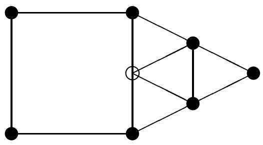
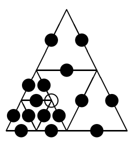
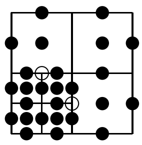
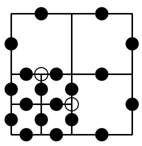
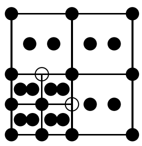
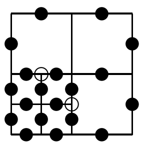
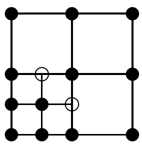
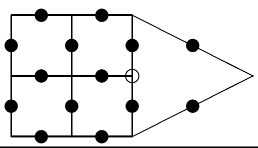
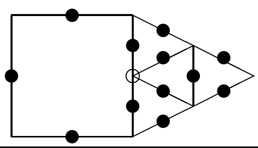
Table 1.1: FEMs with hanging nodes: bullets mark the degrees of freedom and circles mark hanging nodes subsect to internal restrictions.

illustration	reference	illustration	reference
	Courant		Q_1
			
	[27]		[32]
	[37]		Wilson [38]
	[30]		CNR [34]
			

organization of the degrees of freedom is not too easy. Many authors assume a master-slave concept in the sense that there are free nodes and the hanging nodes follow by interpolation. Fig. 1.1 displays some mesh which, in case of continuous discrete functions and vanishing Dirichlet boundary conditions, has not a single free node. Moreover, the concept of parents and children is not immediate here. Hence, at second glance, the concept of hanging nodes is associated with a hierarchy of discretisations and so a sequence of meshes. This is outlined in Section 2 with definitions of concepts like 1-irregular meshes and associated conforming and nonconforming first-order finite element spaces.

Throughout the paper, we discuss five assumptions (A1)-(A5) which we comment very briefly