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COUPLING OF FINITE ELEMENT AND BOUNDARY ELEMENT METHODS FOR THE SCATTERING BY PERIODIC CHIRAL STRUCTURES*

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Dedicated to Professor Junzhi Cui on the occasion of his 70th birthday

Abstract

Consider a time-harmonic electromagnetic plane wave incident on a biperiodic structure in $\mathbf{R}^3$. The periodic structure separates two homogeneous regions. The medium inside the structure is *chiral* and *nonhomogeneous.* In this paper, variational formulations coupling finite element methods in the chiral medium with a method of integral equations on the periodic interfaces are studied. The well-posedness of the continuous and discretized problems is established. Uniform convergence for the coupling variational approximations of the model problem is obtained.

Mathematics subject classification: 65N30, 78A45, 35J20.
Key words: Chiral media, Periodic structures, Finite element method, Boundary element method, Convergence.

1. Introduction

Consider a time-harmonic electromagnetic plane wave incident on a biperiodic structure in $\mathbf{R}^3$. By biperiodic structure or doubly periodic structure, we mean that the structure is periodic in two orthogonal directions. The periodic structure separates two homogeneous regions. The medium inside the structure is chiral and nonhomogeneous. The diffraction problem is to study the propagation of the reflected and transmitted waves away from the structure. Recently, there has been a considerable interest in the study of scattering and diffraction by chiral media. Such media are isotropic, reciprocal, and more importantly circularly birefringent, with potential applications in antennas, microwave devices, waveguides, and many other fields. In general, electromagnetic wave propagation in a chiral medium is governed by Maxwell's equations and a set of constitutive equations known as the Drude-Born-Fedorov constitutive equations, in which the electric and magnetic fields are coupled. The coupling is responsible for the chirality of the medium. It is measured by the magnitude of the chirality admittance β, which along with the dielectric coefficient ε and the magnetic permeability constant μ characterize completely the electromagnetic properties of the medium. On the other hand, periodic (gratings) structures

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have received increasing attentions through the years because of important applications in integrated optics, optical lenses, anti-reflective structures, holography, lasers, communication, and computing. Chiral gratings provide an exciting combination of the medium and structure. The combination gives rise to new features and applications. For instance, chiral gratings are capable of converting a linearly polarized incident field into two nearly circularly polarized diffracted modes in different directions. For an interesting explanation and references of these equations and various physical and computational aspects of the electromagnetic wave propagation inside chiral media, we refer to Lakhtakia [39] and Lakhtakia, Varadan, and Varadan [40] (non-periodic chiral structures), and to Jaggar, *et al.* [38], Lakhtakia, Varadan, and Varadan [41], and Yueh and Kong [55] (periodic chiral structures). Results and additional references on closely related periodic achiral structures may be found in Petit [42] and Bao, Dobson, and Cox [16], Dobson and Friedman [33], Abboud [1], Bao [13], Bao and Dobson [15], Bao and Zhou [18], Chen and Wu [26], Bao, Chen, and Wu [14], Arens, Chandler-Wilde, and DeSanto [12], and Rathsfeld, Schmidt, and Kleeman [51]. Other related recent results for Maxwell's equations in general media may be found in [17, 27, 35, 36].

This paper is devoted to a new approach for solving the diffraction problem, which couples a finite element method (FEM) in the nonhomogeneous chiral medium with a method of integral equations or boundary element method (BEM) on the periodic interfaces. More precisely, the approach consists of two processes: First, a finite element method is used for solving the diffraction problem in the complicated structure of a nonhomogeneous and possibly chiral material. Second, a method of integral equations is developed to derive the exact boundary conditions. The fact that these exact boundary conditions are formulated on the surface of the structure implies that no mesh of the surrounding medium would be needed. In this work, the well-posedness of the continuous and discretized formulations is established. Uniform convergence for the coupling variational approximations of the model problem is obtained. We point out that the variational coupling formulations introduced here are extremely general in terms of material, grating geometry, as well as the incident angle. The material functions ε, μ, and β are only assumed to be bounded measurable. Also, a recent result of Torres [52] indicates that the boundary on which the integral equations are derived needs only be Lipschitz.

Our present coupling approach is related to several other works in the literature. Levillain [43] implemented computationally several versions of a coupling procedure for Maxwell's equations in a three dimensional medium surrounding a bounded perfectly conducting body. de La Bourdonnaye [20] analyzed some coupling formulations for the Helmholtz equation as well as Maxwell's equations. Mathematical analysis of the coupling formulations in [43] has been carried out by Ammari and Nédélec [7, 8]. The results of [7] and [8] are further extended in [9] to study coupling FEM/BEM formulations for Maxwell's equations with a Leontovich boundary condition. We also refer to Wendland [53] and Gatica and Wendland [34] for a survey of asymptotic error estimates for symmetric and nonsymmetric coupling of finite and boundary element methods and to Nédélec [49] for a recent survey of the integral equation methods for computational electromagnetics.

Recently, in [2, 3], the authors have studied mathematical aspects of the diffraction problem by a periodic chiral structure. It is shown that for all but possibly a discrete set of parameters, the diffraction problem attains a unique quasi-periodic weak solution. Our proof is based on a Hodge decomposition lemma along with a new compact imbedding result. An important step of our approach is to reduce the diffraction problem into a bounded domain by using a pair of transparent boundary conditions. The approach in the present paper is different from