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A NONCONFORMING ANISOTROPIC FINITE ELEMENT APPROXIMATION WITH MOVING GRIDS FOR STOKES PROBLEM *1)

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Abstract

This paper is devoted to the five parameters nonconforming finite element schemes with moving grids for velocity-pressure mixed formulations of the nonstationary Stokes problem in 2-D. We show that this element has anisotropic behavior and derive anisotropic error estimations in some certain norms of the velocity and the pressure based on some novel techniques. Especially through careful analysis we get an interesting result on consistency error estimation, which has never been seen for mixed finite element methods in the previously literatures.

Mathematics subject classification: 65N30.
Key words: Stokes problem, Nonconforming finite element, Anisotropy, Moving grids, Error estimate.

1. Introduction

We usually apply the finite element methods to the spatial domain, but choose difference methods with respect to the time variable for solving partial differential equations depending on time. At the same time, different meshes of domain are used at different time level. As we all know, the solutions may have weak regularity at the beginning, therefore, lower order interpolation functions and the smaller meshes should be used. As the time goes on, the regularity of the solution becomes better, the higher order interpolation functions and the larger meshes can be used. That is the main idea of the finite element with moving grids.

Local interpolation error estimations for the finite element methods with moving grids are developed in the literatures [2,3]. But these results are based on isotropic meshes at any time and on any domain. In fact, many examples show that the solutions sometimes have anisotropic behaviors [1,6] on boundary or interior layers. That means that the solution varies significiently only in certain directions. In such cases it is an obvious idea to reflect this anisotropy in the discretization by using anisotropic meshes with a small size in the direction of the rapid variation of the solution and a larger mesh size in the perpendicular direction. That is, anisotropic meshes are necessarily used.

Recently, there have appeared some studies on anisotropic meshes [1,8,14,15,16,18,19], but most of all only considered the interpolation error estimation and conforming elements for elliptic boundary problems. The nonconforming elements and Stokes problem on anisotropic meshes are hardly treated, [9] studied the anisotropic error of Crouzeix-Raviart type elements and applied them to possion problem, [6] studied the quasi-wilson element under a new framework. However, as to anisotropic meshes with moving grids there have been no articles published on

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this respect. Because of the restrictions of the BB conditions, nonconforming finite elements are particularly interested in mixed methods for problems like the velocity-pressure type of Stokes equations, which are advanced in simply structure, economic computing and matching of error orders. The well-known examples include nonconforming Crouzeix-Raviart element [13], the rotated Q_1 element [4] and so on, but these elements can only be used to deal with Stokes equations with moving grids under the regular assumption [10]. [5] developed a kind of nonconforming rectangular element and gave the error estimation for stationary Stokes equations on isotropic meshes. In this paper, we will first show that the element in [5] has anisotropic behavior , and then we derive the error estimation of the stationary Stokes equations on anisotropic meshes. Furthermore, with the idea of moving grids, we study the nonstationary Stokes equations and address the anisotropic nonconforming error estimations based on some results of the stationary problem and some novel approaches. At the same time, by careful analysis we will prove a very interesting and more important result, that is , when the solution $(u,p) \in (H^2(\Omega))^2 \times H^1(\Omega)$ the consistency error is of order $O(h^2)$, one order higher than that of interpolation error which is similar to the reports of [11] for the triangular quasi-conforming and generalized conforming finite elements of fourth order plate bending problem, and the double set 12-parameter rectangular element obtained in [12], and that of [16] for the quasi-wilson element for the second order problems.

Throughout the paper we use the following concerning indices. For the sake of simplity, let $\Omega \subset R^2$ be a rectangular domain with boundary $\partial\Omega$ parallel to x-axis or y-axis. Let Γ_h be a family of rectangular subdivisions, i.e., $\overline{\Omega} = \bigcup\limits_{K \in \Gamma_h} K$. Denote by h_K the diameter of the finite element K, and by ρ_K the supremum of the diameters of all balls contained in K. Then the regularity assumption in the classical finite element theory is $\frac{h_K}{\rho_K} \le C, \forall K \in \Gamma_h$ (The C is a positive constant independent of Γ_h and of the function under consideration). This assumption is no longer valid in the case of anisotropic meshes. Conversely, anisotropic element K is characterized by $\frac{h_K}{\rho_K} \longrightarrow \infty$ where the limit can be considered as $h \longrightarrow 0, h = \max\limits_K h_K$. In this paper the C will also denote the positive constant, not necessarily the same at different occurances which is independent of $\frac{h_K}{\rho_K}$ and h. For the general element K, we denote the lenghths of sides parallel to x-axis and y-axis by $2h_x$ and $2h_y$ respectively, and the central point of K by (x_K, y_K). Let $\widehat{K}$ be a reference element(see Fig.1.), and $\widehat{K} = [-1,1] \times [-1,1]$ with vertices $\widehat{d_1} = (-1,-1), \widehat{d_2} = (1,-1), \widehat{d_3} = (1,1), \widehat{d_4} = (-1,1)$. Let $\widehat{l_1} = \overline{\widehat{d_1}\widehat{d_2}}, \widehat{l_2} = \overline{\widehat{d_2}\widehat{d_3}}, \widehat{l_3} = \overline{\widehat{d_3}\widehat{d_4}}, \widehat{l_4} = \overline{\widehat{d_4}\widehat{d_1}}$ be the four sides of $\widehat{K}$. The transformation of $F_K : \widehat{K} \longrightarrow K$ is defined by

$$\begin{cases} x = x_K + h_x \xi, \\ y = y_K + h_y \eta. \end{cases} \tag{1.1}$$

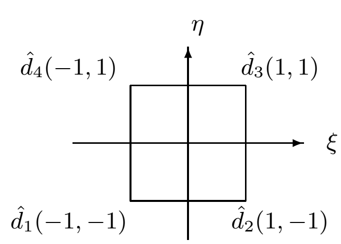

Fig.1. the reference element $\widehat{K}$