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RECENT ADVANCES OF UPSCALING METHODS FOR THE SIMULATION OF FLOW TRANSPORT THROUGH HETEROGENEOUS POROUS MEDIA *1)

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Dedicated to the 70th birthday of Professor Lin Qun

Abstract

We review some of our recent efforts in developing upscaling methods for simulating the flow transport through heterogeneous porous media. In particular, the steady flow transport through highly heterogeneous porous media driven by extraction wells and the flow transport through unsaturated porous media will be considered.

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1. Introduction

The central difficulty in the modeling of subsurface flow and transport is the accounting for the spatial variability in the parameters used to characterize the relevant physical properties of the natural porous media. In realistic situations, the precise spatial distribution of the parameters required to characterize the problem is never available due to the lack of enough data. Thus sophisticated geological and geostatistical modeling tools are used in practice to generate highly detailed medium parameters based on some site-specific measurements and experience from other sites. There exists a vast literature on the upscaling or homogenization techniques that lump the small-scale details of the medium into a few representative macroscopic parameters on a coarse scale which preserve the large-scale behavior of the medium and are more appropriate for reservoir simulations. We refer to the book of Christakos [7] for more information on the random field modeling of the natural porous medium parameters and the recent review paper [24] on the existent upscaling techniques in the engineering literature.

The recently introduced multiscale finite element method [15, 16] for solving elliptic equations with oscillating coefficients provides an effective way to capture the large scale structures of the solutions on a coarse mesh without resolving all the fine scale structures. The central idea of the method is to incorporate the local small scale information of the leading order differential operator into the finite element bases. It is through these multiscale bases and the finite element formulation that the effect of small scales on the large scales is correctly captured. We also refer to the related analysis of the heterogeneous multiscale method (HMM) [19] for solving the elliptic problem with oscillating coefficients. In section 2 we will describe one engineering upscaling technique and discuss its relation with the multiscale finite element method.

The study of steady flow through highly heterogeneous porous medium driven by extraction wells is of great importance in hydrology, petroleum reservoir engineering. It is observed in the

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engineering literature (cf. e.g. [8] and [21] and the references therein) that in the near-well region, many of the existing upscaling methods do not provide satisfactory results. The reason may be explained as the standard upscaling methods generally assume the pressure field is slowly varying, that is clearly not true in the vicinity of the flowing wells [8]. This fact may also be explained mathematically from the homogenization theory for the second order elliptic equations with periodic coefficients. In the homogenization theory, multiscale convergence is ensured under the assumption that the source should be at least in H^{-1} so that the solution is bounded in Sobolev space H^1. As we will see below, however, well singularities can be modeled as Dirac sources and thus in the near-well region, the solution behaves like Green function which is not uniformly bounded in H^1. In section 3 we will describe an upscaling technique for dealing with well singularities.

The nonlinear Richards equation which models the flow transport in unsaturated porous media is of significant importance in engineering applications. We consider the following nonlinear partial differential equations

$$\frac{\partial\theta}{\partial t} - \frac{\partial K}{\partial x_3} - \nabla\cdot(K\nabla u) = f,$$

where θ is volumetric water content, K is the absolute permeability tensor, u is the fluid pressure, x_3 denotes the vertical coordinate in the medium, and f stands for possible sources/sinks. The sources of nonlinearity of Richards equation come from the moisture retention function $\theta(u)$ and relative hydraulic conductivity function $K(\theta)$, respectively. Based on experimental results, many different functional relations have been proposed in the literature through various combinations of the dependent variables θ, u and K, and a certain number of fitting parameters (e.g., [13, 14]). There are several widely known formulations of the constitutive relations such as the van Genuchten-Mualem model [14], or the Garder model [13]. For example, in the Garder model, also called exponential model,

$$\theta(u) = \theta_r + (\theta_s - \theta_r)e^{-\beta|u|}, \quad K(u) = K_s e^{-\alpha|u|},$$

where θ_r and θ_s represent the residual water content and saturated water content respectively, K_s is the saturated hydraulic conductivity, and α, β are parameters of the porous media. In section 4 we develop an upscaling method for a class of nonlinear parabolic equations which includes the Richards equation in the parabolic range as a special case.

2. Upscaling of the Permeability

The purpose of this section is to show that one of the well-known engineering upscaling techniques (see e.g. [20]) is equivalent to the multiscale finite element method proposed in [10, 15]. We remark that multiscale finite element method is shown to be convergent under the condition that the permeability is locally periodic $K_\varepsilon(x) = K(x, x/\varepsilon)$, where $K(x,\cdot)$ is periodic with respect to the second variable. As a consequence, the convergence of the engineering approach described in this section is guaranteed.

Let $\mathcal{M}_H$ be a finite element mesh of Ω with the mesh size H much larger than the ε, the characteristic length representing the small scale variability of the media. Usually, ε is equal to the correlation length in the statistical random field modeling of the media. Let W_H be the standard conforming linear finite element space over $\mathcal{M}_H$ and $W_H^0 = W_H \cap H_0^1(\Omega)$. In the engineering literature, the problem

$$-\mathrm{div}\,(K_\varepsilon(x)\nabla u_\varepsilon) = f \quad \text{in } \Omega, \quad u_\varepsilon = 0 \quad \text{on } \Gamma \tag{2.1}$$

is approximated by the homogenized or upscaled problem: Find $u_H^* \in W_H^0$ such that

$$\int_\Omega K^*(x)\nabla u_H^* \nabla v_H dx = \int_\Omega f v_H dx \quad \forall v_H \in W_H^0 \tag{2.2}$$

with K^* being piecewise constant on the coarse mesh $\mathcal{M}_H$. The so-called effective permeability matrix K^* on each $T \in \mathcal{M}_H$ is defined as follows. For any $G \in \mathbf{R}^2$, let θ_ε be the solution of the problem

$$-\mathrm{div}\,(K_\varepsilon \nabla\theta_\varepsilon) = 0 \text{ in } T, \quad \theta_\varepsilon|_{\partial T} = G\cdot x.$$