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A SMOOTHING LEVENBERG-MARQUARDT TYPE METHOD FOR LCP [*1)]

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Abstract

In this paper, we convert the linear complementarity problem to a system of semismooth nonlinear equations by using smoothing technique. Then we use Levenberg–Marquardt type method to solve this system. Taking advantage of the new results obtained by Dan, Yamashita and Fukushima [11, 33], the global and local superlinear convergence properties of the method are obtained under very mild conditions. Especially, the algorithm is locally superlinearly convergent under the assumption of either strict complementarity or certain nonsingularity. Preliminary numerical experiments are reported to show the efficiency of the algorithm.

Mathematics subject classification: 65K05, 90C30, 90C31, 90C33.
Key words: LCP, Levenberg–Marquardt method, Smoothing technique, P_0 matrix, Superlinear convergence.

1. Introduction

Consider the following linear complementarity problem (LCP)

$$\begin{gathered} y = Mx + q, \\ x \geq 0,\ y \geq 0,\ x^T y = 0, \end{gathered} \tag{1}$$

where $M \in R^{n\times n}$, $x, y \in R^n$ and $x \geq 0$ means that $x_i \geq 0$ $(i = 1, \ldots, n)$. In this paper, we assume that the solution set of (1) is nonempty. Let X denote the solution set of (1).

LCP has many applications in economic and engineering, see [16] for a survey. A few experts have studied the problem and numerous algorithms were proposed for the problem, for examples, interior methods (see [37] and references therein), nonsmooth Newton methods (see [12, 15]) and smoothing methods (see [27] and references therein). Among these methods, some algorithms are superlinearly (quadratically) convergent under one of the following group of conditions:

(i). M is P matrix and one of the cluster points of the sequence generated by the algorithm is nondegenerate;

(ii). M is P_0 matrix and certain nonsingularity is satisfied at one of the cluster points of the sequence generated by the algorithm.

So we ask whether there exists a method which is superlinearly convergent without assumption of strict complementarity and nonsinguarity at the limit point. In this paper, we give a

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method which is superlinearly convergent under the assupmtion of that M is P_0 and either strict complementarity or nonsingularity.

As we known, only the method in [21] is convergent superlinearly/quadratically under the assumptions of either that M is P matrix and strict complementarity or that M is P_0 and certain nonsingularity. Our method is convergent superlinearly under each of the following group conditions simultaneously: (i). M is P_0 matrix and one of the cluster points of the sequence generated by the algorithm is strict complementarity; (ii). M is P_0 matrix and certain nonsingularity is satisfied at one of the cluster points of the sequence generated by the algorithm. So the results in this paper is stronger than the results in [21].

Levenberg–Marquardt method (LMM) is a classical algorithm for solving the following system of nonlinear equations

$$F(x) = 0,$$

where F is a mapping from R^n to R^m. At each iterative point x_k, the search direction is obtained by solving the following linear equation system

$$(F'(x_k)^T F'(x_k) + \mu_k I)d = -F'(x_k)^T F(x_k),$$

where $F'(x)$ denotes the Jacobian of $F(x)$ and $\mu_k > 0$ is a parameter. It is well known that the method is superlinearly convergent if μ_k is updated by an appropriate rule and certain nonsingularity of $F'(x)$ is satisfied at a limit point. Recently, Yamashita and Fukushima [33], Dan, Yamashita and Fukushima [11] proposed a new update rule for μ_k, i.e., $\mu_k = \|F(x_k)\|^\delta$ and proved that LMM is superlinearly(quadratically) convergent under local error bound condition without nonsingularity. Then they applied their method to the linear complementarity problem and obtained an algorithm which has superlinear (quadratical) convergence properties under the conditions that M is P_0 matrix and there exists a cluster point being nondegenerate. This result is very interesting.

In this paper, we convert LCP into a system of semismooth nonlinear equations $F(x, y, \tau) = 0$ by using smoothing technique and by viewing the smoothing parameter as an independent variable. Then LMM type algorithm is proposed to solve this semismooth system. It is similar to [11, 33] that $\mu^k = \|F(x^k, y^k, \tau^k)\|^\delta$. We can prove that our algorithm is globally convergent and the algorithm is superlinealy (quadratically) convergent under the assumption of either strict complementarity or certain nonsingularity. Note that there is an essential difference between our algorithm and the ones in [11, 33] since here we shall keep $\tau_k > 0$ at each iterative point $({x^k}^T, {y^k}^T, \tau^k)^T$. Therefore, our algorithm is not a simple application of Yamashita and Fukushima's algorithm.

Now we explain our notations. Throughout the paper, all of the vectors are column vector. R^n_+ denotes n-dimensional nonnegative orthant, i.e., $x \in R^n_+ \iff x_i \geq 0, i = 1, \ldots, n$, and R^n_{++} denotes the n-dimensional positive orthant, i.e., $x \in R^n_{++} \iff x_i > 0, i = 1, \ldots, n$. Sometimes we use (w, τ) for $(w^T, \tau)^T$ and $w = (x^T, y^T)^T$. $\|\cdot\|$ denotes 2-norm and $\|\cdot\|_\infty$ denotes ∞-norm.

The paper is organized as follows. In Section 2, we give some basic results on smoothing reformulation. The algorithm model and its global convergence are stated in Section 3. In Section 4, we show the local convergence properties of the algorithm. In Section 5, some preliminary numerical results are reported. In section 6, some discussions and conclusions are given.

2. Some Basic Results

Let $\psi : R^2 \to R$ be Fisher-Burmeister function

$$\psi(a, b) = a + b - \sqrt{a^2 + b^2}.$$