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A TRULY GLOBALLY CONVERGENT FEASIBLE NEWTON-TYPE METHOD FOR MIXED COMPLEMENTARITY PROBLEMS *1)

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Abstract

Typical solution methods for solving mixed complementarity problems either generate feasible iterates but have to solve relatively complicated subproblems such as quadratic programs or linear complementarity problems, or (those methods) have relatively simple subproblems such as system of linear equations but possibly generate infeasible iterates. In this paper, we propose a new Newton-type method for solving monotone mixed complementarity problems, which ensures to generate feasible iterates, and only has to solve a system of well-conditioned linear equations with reduced dimension per iteration. Without any regularity assumption, we prove that the whole sequence of iterates converges to a solution of the problem (truly globally convergent). Furthermore, under suitable conditions, the local superlinear rate of convergence is also established.

Mathematics subject classification: 65H10.
Key words: Mixed complementarity problems, Newton-type methods, Global convergence, Superlinear convergence.

1. Introduction

We consider the mixed complementarity problem, MCP for simplicity: Find a vector $x^* \in [l,u]$, such that

$$\begin{aligned} x_i^* = l_i \quad &\Rightarrow \quad F_i(x^*) \ge 0, \\ x_i^* \in (l_i, u_i) \quad &\Rightarrow \quad F_i(x^*) = 0, \\ x_i^* = u_i \quad &\Rightarrow \quad F_i(x^*) \le 0, \end{aligned} \tag{1}$$

where $l_i \in R \cup \{-\infty\}$ and $u_i \in R \cup \{+\infty\}$ are given lower and upper bounds with $l_i < u_i$, $i = 1, \cdots, n$, F is a continuously differentiable mapping from the rectangle $[l,u]$ to R^n. The MCP (1) can also be written as the closed form of the variational inequality problem of finding a vector $x^* \in [l,u]$, such that

$$(x - x^*)^\top F(x^*) \ge 0, \quad \forall x \in [l,u]. \tag{2}$$

When $l_i = 0$ and $u_i = +\infty$ for all $i = 1, \cdots, n$, MCP reduces to the nonlinear complementarity problem of finding a vector $x^* \in R^n$, such that

$$x^* \ge 0,\ F(x^*) \ge 0, \quad {x^*}^\top F(x^*) = 0,$$

and if $l_i = -\infty$ and $u_i = +\infty$ for all $i = 1, \cdots, n$, MCP reduces to the nonlinear system of equations

$$F(x) = 0.$$

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The mixed complementarity problem and the nonlinear complementarity problem have a number of important applications in operations research, engineering problems and economics equilibrium problems, see the survey papers [6, 8, 10] for detailed examples and the references therein.

There are many iterative methods for solving the mixed complementarity problem [2, 7, 11, 12, 13, 14, 15, 19, 30]. While the projection-type methods [11, 12, 13, 14, 15] are attractive for its simplicity and global convergence, the most successful and widely used are Newton-type methods. A class of Newton-type methods are Josephy-Newton methods based on solving a series of linear complementarity problems [3, 10, 17, 30]. Given a point x^k, the Josephy-Newton methods generate the next iterate x^{k+1} by solving the following linear mixed complementarity problem

$$\text{Find } x \in [l,u], \text{ such that} \quad F^k(x)^\top(z-x) \geq 0,\ \forall z \in [l,u], \tag{3}$$

where $F^k(\cdot)$ is the first order approximation of F at x^k,

$$F^k(x) = F(x^k) + F^{'}(x^k)(x - x^k) \tag{4}$$

assuming $F(\cdot)$ is differentiable.

Another class of Newton-type method are equation-based reformulation for MCP [18, 22, 32, 33, 36]. At each iteration, this type of methods solves a linear system of equations

$$H_k d^k = -\Phi(x^k) \tag{5}$$

to find the search direction d^k, where Φ is a semismooth function with the property that

$$x \text{ solves MCP (1)} \Longleftrightarrow \Phi(x) = 0,$$

and $H_k \in \partial\Phi(x^k)$ is an element of the generalized Jacobian of Φ at x^k in the Clarke's sense [4]. Note that at each iteration, the equation-based Newton-type methods solve a linear system of equations (5), which is structurally easier to solve than linear mixed complementarity problem (3). Because of the extreme efficiency in practice, this class of Newton-type methods, especially those methods based on Fisher-Burmeister function are recently studied extensively [7, 18, 21, 22, 36, 38]. However, the generated sequence $\{x^k\}$ is not necessarily contained in the feasible set $[l,u]$. At the same time, the feasibility issue for MCP (1) is always important because some real-life applications such as in engineering design and economics [16, 29] require the data only defined in the feasible region. Hence, as mentioned in [21], "it would be extremely nice to have an algorithm that, on the one hand, generates only feasible iterates and, on the other hand, has to solve only simple subproblems". Nevertheless, there are currently only a few methods with these two properties available [21, 24, 38, 39].

A common difficulty with using the Newton-type method by solving (3) or (5) is that, while possessing fast local convergence property, there are serious problems with ensuring global convergence. To enlarge the domain of convergence of the Newton method, many globalization strategy for (3), (5) are proposed. The most natural globalization strategy is a line search procedure in the obtained Newton direction aimed at decreasing the value of some valid merit functions. However, these strategies can only ensure that the generated sequence converges to a stationary point of the merit function, which is a solution of MCP under some restrictive assumptions. These assumptions imply the boundedness of level sets of the merit function and possible uniqueness of the solution. Moreover, some of the merit functions, such as those based on the natural residual [27] and the normal map [34] are nondifferentiable, which make the line search difficult to implement. The differentiable merit functions, such as the gap function [23], the regularized gap function [9] and the D-gap function [30], are designed for special variational inequality problems and/or complementarity problems, and thus each of these globalizations has certain drawbacks. Recently, Solodov and Svaiter [35] proposed a truly globally convergent