Journal of Computational Mathematics, Vol.22, No.2, 2004, 148–155.

RECENT MATHEMATICAL STUDIES IN THE MODELING OF OPTICS AND ELECTROMAGNETICS *1)

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Dedicated to Professor Zhong-ci Shi on the occasion of his 70th birthday

Abstract

This work is concerned with mathematical modeling, analysis, and computation of optics and electromagnetics, motivated particularly by optical and microwave applications. The main technical focus is on Maxwell's equations in complex linear and nonlinear media.

Mathematics subject classification: 65N15, 65N30, 78A45, 78A25
Key words: Maxwell's equations, Optics, Nonlinear optics, Electromagnetics.

1. Micro-optics

Micro diffractive optics is a fundamental and vigorously growing technology which continues to be a source of novel optical devices. Significant recent technology developments of high precision micromachining techniques have permitted the creation of gratings (periodic structures) and other diffractive structures with tiny features. Current and potential application areas include corrective lenses, microsensors, optical storage systems, optical computing and communications components, and integrated opto-electronic semiconductor devices. Because of the small structural features, light propagation in micro-optical structures is generally governed by diffraction. In order to accurately predict the energy distributions of an incident field in a given structure, the numerical solution of full Maxwell's equations is required. Computational models also allow the exciting possibility of obtaining completely new structures through the solution of optimal design problems. General discussion and recent advances on the diffraction problem may be found in [32], [19], [16] and references therein.

1.1. Computation and Analysis of the Diffraction Problem

The diffraction problem is to predict the electromagnetic field distributions when a time-harmonic plane wave is incident on a given grating or periodic structure. Because of the small structural features, wave propagation is dominated by the system of Maxwell's equations. Here, the diffractive structure is periodic either in one direction – linear grating (2-D model) or in two orthogonal directions – biperiodic structure (3-D model). In the two-dimensional case, convergence analysis for the finite element methods in TE (transverse electric) and TM (transverse magnetic) polarization was first carried out in Bao [11] and [12]. In both cases, existence and uniqueness for the continuous and discreet model problems were established. In general, to model the diffraction of biperiodic diffractive structures, *i.e.,* a three-dimensional geometry, it is essential to study Maxwell's equations in vector form. In Bao and Dobson [18],

* Received January 31, 2004.
1) The research of the author was partially supported by the NSF grant DMS 01-04001 and the ONR grant N000140210365.

a new variational formulation for the diffraction problem in biperiodic structures (3D) was introduced. Using the formulation, we established the well-posedness of the weak solutions (the magnetic fields) in H^1. In contrast, the classical approach only gives rise to the existence and uniqueness of $H(curl)$ solutions. This presents a severe difficulty in numerical analysis since the imbedding from $H(curl)$ to L^2 is not compact. It was established in [27] the well-posedness of the discretized problem. Error estimates for the variational (finite element) approximation of the model problem was obtained. These convergence estimates are quite general since little smoothness is assumed on the coefficients and the geometry.

A related project is on least-squares finite element analysis of diffraction problems. In the two-dimensional case, a least-squares finite element method was first proposed in [26] that incorporates the jump conditions at interfaces into the objective functional. Optimal error estimates were obtained. The results indicate that significantly better error estimates than standard finite element methods may be obtained for sufficiently smooth interfaces. More recently, for the two-dimensional diffraction problem, Chen and Wu [30] have developed a new approach by combining the adaptive finite element method and a perfectly matched layer (PML) boundary condition.

1.2. Chiral Gratings

Chiral gratings provide an exciting combination of the medium and structure, which gives rise to new features and applications. For instance, chiral gratings are capable of converting a linearly polarized incident field into two nearly circularly polarize diffracted modes in different directions. Mathematically, in a chiral medium, the Maxwell equations remain the same form. However, the constitutive equations are now coupled. Therefore, the model equations are necessarily of vector form and much more complicated than standard Maxwell's equations. A variational approach was developed by Ammari and Bao [1]. We established the existence and uniqueness of weak solutions by a combination of a variational approach and the Hodge decomposition of the electric field.

1.3. Scattering by a Perturbed Periodic Structure

Consider a time-harmonic electromagnetic plane wave incident on a periodic (grating) structure. An inhomogeneous (subwavelength) object is placed inside the periodic structure. The scattering problem is to study the electromagnetic field distributions. The problem arises in the study of near-field optics and has many physical and biological applications. In [2], we developed an integral representation approach to solve the model problem. It was shown particularly that the perturbation due to the object decays exponentially along the periodic direction of the structure, provided that no surface waves occur. Based on the approach, a general solution method may be introduced. More recently, the approach has been generalized to study the three dimensional Maxwell equation model [4].

1.4. Inverse Diffraction and Optimal Design Problems

Consider scattering of electromagnetic waves by a doubly periodic structure. Above the structure, the medium is assumed to be homogeneous with a constant dielectric coefficient. The medium is a perfect conductor below the structure. An inverse problem arises and may be described as follows [10]. For a given incident plane wave, the tangential electric field is measured away from the structure. To what extent can one determine the location of the periodic structure that separates the dielectric medium from the conductor? This inverse problem which arises naturally in the optimal design of gratings has received much attention recently. However, most of the progress has been made only in the 2-D or scalar case where the structure and material are assumed to be invariant in one direction. In this 2-D setting, Friedman and Bao obtained by using a variational method and index theory in [21] the first set of stability results for a large class of inverse diffraction problems. The result indicates that for small h, if the