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A TRUST REGION METHOD FOR SOLVING DISTRIBUTED PARAMETER IDENTIFICATION PROBLEMS *1)

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Abstract

This paper is concerned with the ill-posed problems of identifying a parameter in an elliptic equation which appears in many applications in science and industry. Its solution is obtained by applying trust region method to a nonlinear least squares error problem. Trust region method has long been a popular method for well-posed problems. This paper indicates that it is also suitable for ill-posed problems. Numerical experiment is given to compare the trust region method with the Tikhonov regularization method. It seems that the trust region method is more promising.

Key words: Parameter identification, Ill-posed problems, Trust region.

1. Introduction

Parameter identification problems play an important role in many applications in science and industry (see [1, 3]). By parameter identification, we refers that the estimation of coefficients in a differential equation from observations of the solution to that equation. We call the coefficients the system parameters, and the solution and its derivatives the state variables. The forward problem is to compute the state variables given the system parameters and appropriate boundary conditions, which is a well-posed problems. However in parameter identification, the problems is typically ill-posed (see [5]).

For example, we consider the problem of identifying a distributed parameter $q = q(x)$ in the one-dimensional steady-state diffusion equation in the form

$$-\nabla(q\nabla u) = g, \text{ in } (0,1) \tag{1}$$

with Dirichlet boundary conditions

$$u(0) = u_0, \ u(1) = u_1.$$

This is used to model for example, the steady-state temperature distribution within a thin metal rod (see [12]). Another example is the inverse groundwater filtration problem of reconstructing the diffusivity q of a sediment from measurements of the piezometric head u in the steady state case (see [1] for further applications). We take the former case as our example. In this kind of setting, the state variable is the temperature distribution $u(x)$, $x \in (0,1)$, the system parameters are diffusion coefficient $q(x)$ and the heat source term $g(x)$. The inverse problems stated here is determining parameter $q(x)$ by giving $g(x)$ and $u(x)$ for $x \in [0,1]$.

For sake of simplifying the notations, we outline the problem in the abstract operator form

$$F(q)u = g, \tag{2}$$

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where $F(q)$ represents a parameter-dependent differential operator from the parameter space Q to the state space U, $q \in Q$ represents the distributed parameter to be estimated, and $u \in U$ represents the corresponding state variable. In case of the above example, q represents the diffusion coefficient, and

$$F(q) = -\nabla(q\nabla(\cdot)).$$

Since u is the observation data, therefore, it may contain noise. Assume that the observed data can be expressed as

$$u_e = u + e \tag{3}$$

with Gaussian noise e.

Because of the ill-posedness of the problem (1), some kind of regularization technique has to be applied (see [5, 13, 24]). Perhaps Tikhonov regularization method (see [9, 20]) is the most well-known method for dealing with such kind of problems.

Given the regularization parameter $\alpha > 0$, choose $q^\alpha \in Q$ to solve the unconstrained minimization problem

$$\min_{q \in Q} M^\alpha[q] := \|F(q)u_e - g\|^2 + \alpha\|q\|^2, \tag{4}$$

where $\alpha > 0$ is called the regularization parameter and $\|q\|^2$ serves as the stabilizer.

Assume the forward problem solving for u is well-posed, then we can denote the solution by

$$f(q) := u = F^{-1}(q)g. \tag{5}$$

Clearly we want to minimize the following constrained functional

$$J_{q\in Q}(q) = \frac{1}{2}\|u - u_e\|^2, \tag{6}$$

$$s.\ t.\ F(q)u = g. \tag{7}$$

By (5), problem (6)-(7) is equivalent to the unconstrained regularized least squares minimization problem

$$\min J_{q\in Q}(q) = \frac{1}{2}\|f(q) - u_e\|^2. \tag{8}$$

Certainly we can use the Tikhonov regularization to (5), for which, we have the following minimization problem:

$$\min J_{q\in Q}(q) = \frac{1}{2}\|f(q) - u_e\|^2 + \alpha\theta(q), \tag{9}$$

where $\theta(q)$ is a regularized functional whose duty is to impose stability, $\alpha > 0$ is a regularization parameter.

This paper will deal with the problem in a different way: i.e., we use some kind of approximation to the original problem (8), then the trust region technique is used.

2. Finite Dimensional Approximation: Trust Region Method

First we introduce the trust region method in a general way. Trust region methods are a group of methods for ensuring global convergence while retaining fast local convergence in optimization algorithms. For example, we consider the minimization problem

$$\min_{x\in\mathcal{R}^n} f(x). \tag{10}$$