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HIGH ACCURACY NUMERICAL METHOD OF THIN-FILM PROBLEMS IN MICROMAGNETICS*1)

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Dedicated to the 80th birthday of Professor Zhou Yulin

Abstract

In this paper, a new high accuracy numerical method for the thin-film problems of micron and submicron size ferromagnetic elements is proposed. For the computation of stray field, we use the finite element method(FEM) by introducing a semi-discrete artificial boundary condition [1, 2]. In our numerical experiments about the domain patterns and their movement, we can see that the results are accordant to that of experiments and other numerical methods. Our method are very convenient to deal with arbitrary shape of thin films such as a polygon with high accuracy.

Key words: Thin-film, Micromagnetics, stray field, Semi-discrete artificial boundary condition.

1. Introduction

Micromagnetism of micron and submicron scale patterned thin-film has become an area of great scientific and technological interest in recent years[3, 4, 5, 6, 7]. Because of the important applications of ferromagnetic thin-film to magnetic information storage technology and the potential of the semiconductor microelectronics technology, there has been a rising interest in studying the efficient numerical methods for the thin-film problems in the world[5, 6, 7].

One can simulate the magnetization processes by combining the classic micromagnetic theory with dynamic descriptions of magnetization orientations. Micromagnetic theory considers the free energy in the ferromagnetic material, which in general includes the following energy terms (here we omit the *magnetoelastic energy*)

1. the *magnetic anisotropy energy*, which acts as a local constraint on the magnetization orientation,
2. the *exchange energy*, which tends to keep adjacent spins parallel,
3. the *magnetostatic (or Self-Induced) energy*,
4. the *magnetic potential energy* due to external magnetic fields,

By the simulation of the dynamic process in Micromagnetic modelling, not only we can get the remanence domain configurations in a ferromagnetic element, but also we can get the transient pictures that demonstrate how a complex domain structure forms. From the Micromagnetic Model we know that the complex magnetization domain patterns and the detailed spin structures within the domain boundaries are the results of minimizing the total free energy. That is, the different domain patterns correspond to different local energy minima. If an external field

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is applied with sufficient magnitude that the energy minimum disappears, the corresponding magnetization domain pattern will change, following the dynamic equation until a new energy minimum is reached.

In this paper, we provide a method which mixed the finite element method and integral method to get the numerical solution of the ferromagnetic thin-film problem. First, we rewrite the solution of an initial-boundary problem with the Landau-Lifshitz equation in integral formula, we can reduce the computation of the most singular part of the integral to a Poisson problem on an infinite domain in two dimensional (2D) [7]. After that, we can design a semi-discrete artificial boundary condition [1, 2] to get the numerical solution by finite element method (FEM).

2. Thin-film problem

One class of ferromagnetic thin films that has been studied extensively by micromagnetic modelling are the magnetic thin films used for data storage in hard-disk drives. In general, these thin films are a few tens of nanometers thick and less than a micron long. Therefore, we will focus on the micron and sub-micron size thin-films with tens nanometers thickness in this paper. Certainly, our method can deal with more general thin-film problems.

First, let's recall the full micromagnetic model [5, 7, 8]. Consider a ferromagnetic material contained in a domain $V_\delta = \Omega \times [-\delta, \delta] \subset \mathbb{R}^3$, where $\delta << diam(\Omega)$, $\Omega \subset \mathbb{R}^2$ is supposed to have a piecewise smooth boundary, for example, a polygon (then we can define the out normal vector on $\partial\Omega$ except a finite number of points). As we mentioned, the free-energy functional of micromagnetics can be written as

$$\begin{aligned} E(\mathbf{m}) &= \frac{\mathcal{A}}{2}\int_{V_\delta} |\nabla \mathbf{m}|^2 dx + \frac{\mathcal{K}_\mu}{2}\int_{V_\delta} \phi(\mathbf{m})\, dx + \frac{M_s^2}{2\mu_0}\int_{\mathbb{R}^3} |\mathbf{h}_{str}|^2 dx - \frac{M_s^2}{\mu_0}\int_{V_\delta} \mathbf{h}_{ext}\cdot \mathbf{m}\, dx \\ &\equiv E_{exc} + E_{ani} + E_{sta} + E_{ext}, \end{aligned} \tag{2.1}$$

where E_{exc}, E_{ani}, E_{sta} and E_{ext} represent *exchange energy*, *anisotropy energy*, *static energy*, and *external field energy* respectively. Here M_s is saturation magnetization, $\mathbf{m}$ is the normalized magnetization (= the magnetization $\mathbf{M}/M_s$), a unit vector field defined on the film V_δ. Moreover, $\mathcal{A}$ (dimension J/m) is the exchange stiffness constant, measures the strength of the exchange energy relative to that of dipolar interactions, $\mathcal{K}_\mu$ (dimension J/m^3) is the quality factor measuring the relative strength of the magnetic anisotropy ϕ, $\mathbf{h}_{str}$ is the normalized stray field, whose norm squared gives the magnetostatic energy density, $\mathbf{h}_{ext}$ is the applied field, which we assume to be uniform.

The effective magnetic field $\mathbf{h}_{eff}$ at a position inside the ferromagnetic material is defined by

$$\mathbf{h}_{eff} = -\frac{\delta E}{\delta \mathbf{m}}. \tag{2.2}$$

The magnetization orientation follows the Landau-Lifshitz equation[8],

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m}\times \mathbf{h}_{eff} - \alpha\gamma \mathbf{m}\times(\mathbf{m}\times \mathbf{h}_{eff}), \tag{2.3}$$

where γ is the electron gyromagnetic ratio and α is the damping constant. If we want to get the domain patterns or observe the movement of the domain walls, we should solve the following initial-boundary value problem

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m}\times \mathbf{h}_{eff} - \alpha\gamma \mathbf{m}\times(\mathbf{m}\times \mathbf{h}_{eff}) \tag{2.4}$$

$$\mathbf{m}(0,x) = \mathbf{m}_0(x) \tag{2.5}$$